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Theoretical and model analyses of the bridge as a space structure with one concentrated load on one truss

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THEORETICAL AND MODEL ANALYSES
OF THE
BRIDGE AS A SPACE STRUCTURE
WITH
ONE CONCENTRATED LOAD ON ONE TRUSS

By

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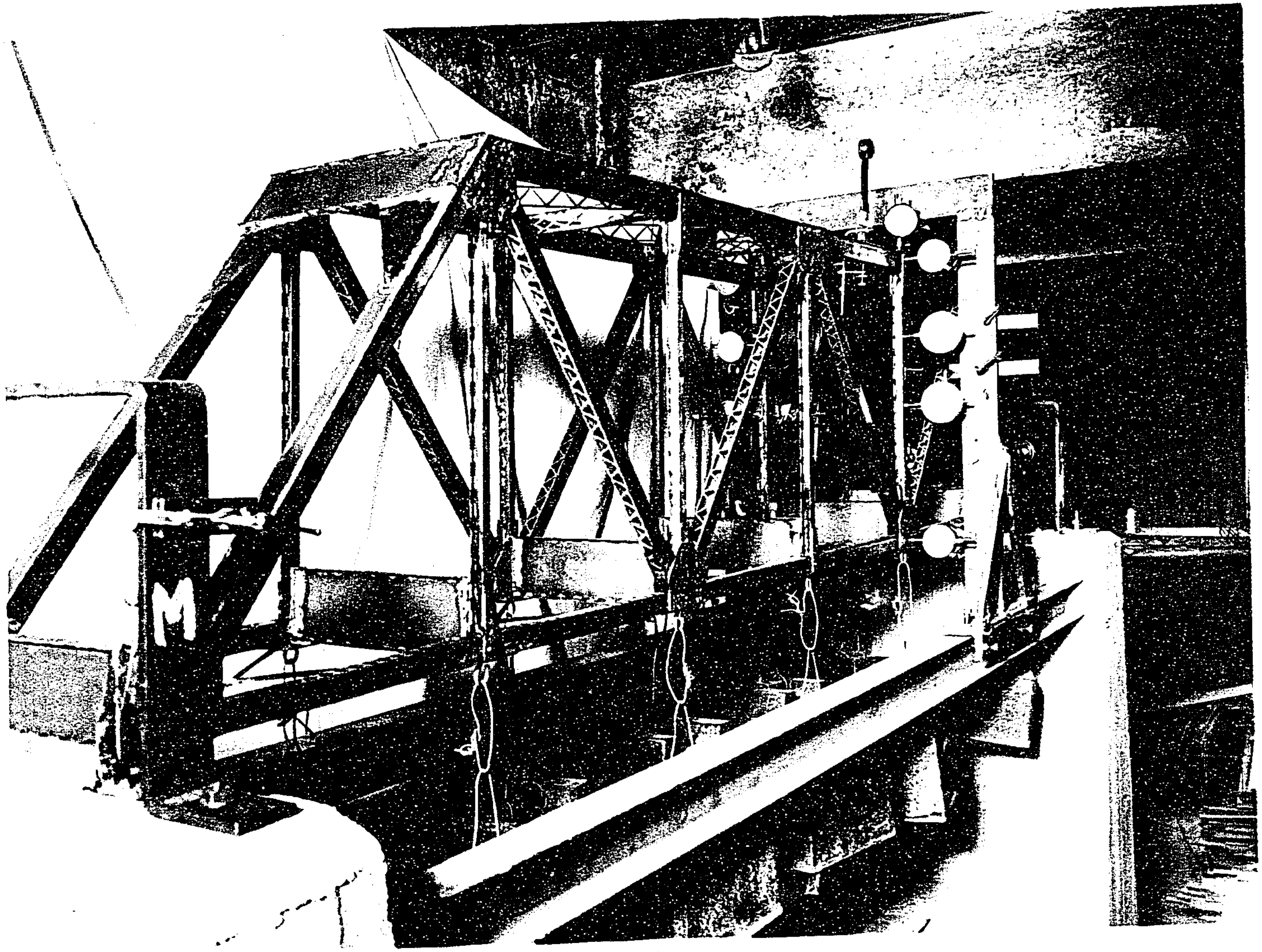

PLATE I.

MODEL OF 150 FOOT SINGLE TRACK RAILROAD BRIDGE

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I. INTRODUCTION

A. SYNOPSIS

This paper deals with the complete analysis of a six panel through railway bridge considered as a space structure. One concentrated load is applied on one vertical truss only, and is unsymmetrically placed on that truss. The theoretical analysis is based upon assumptions which, the author believes, are rational.

A space model is then designed for the same bridge by the Principle of Similitude, and a series of tests are made therewith to check the computed results.

It is possible to obtain an apparently correct solution for the bridge as a space structure with all members pin-connected when full diagonal bracing is assumed at the end panels. Since a number of investigators have considered this solution to be correct, to record the error of this method of analysis, a complete solution is provided in the Appendices.

B. HISTORICAL REVIEW

The fundamental principles of the theory of structures were not very definitely established prior to 1880; the period from 1865 to 1880 marked the most rapid development (27). Thus in 1864, Maxwell published his analysis of redundant frameworks; in 1874 Mohr presented the same kind of analysis which later became generally known as the Maxwell Mohr method of deflections.

In 1868 Winkler formulated his theory of arches; in 1872 Green presented his method of moment areas; and in 1877 Williot discovered the graphical solution for truss deflections. Castigliano published his treatise of least work in 1879; Manderla presented his solution of secondary stresses in 1879; and Winkler developed another method for secondary stress solution in 1880 (1), (27).

Muller-Breslau and Foppl made early contributions to the solution of the stresses in both determinate and indeterminate space structures (27), (48).

Little progress has been made along the analysis of space structures until the last fifteen years, when the precise design of aeroplane structures called for further studies. In 1920, R. V. Southwell presented his method of tension coefficients for space structures, in which all stresses in different members are expressed in terms of their lengths and tension factors (37). In the same year, Southwell has presented a complete solution for redundantly braced frameworks stressed by flexure, torsion, and shear (38). His solution is based on an ideal application of loads.

Also in 1920, Professor A. J. S. Pippard of Bristol, England, and W. D. Douglas published their paper, "Torsional Stresses in the Fuselage of an Aeroplane" (28). In this analysis, the authors, applying the principle of least work, computed the amount of torque distribution among different portions of the structure; the torsion end was assumed to be a rigid plane, and the four corners of the other end were pinned to the wall.

In 1924, Professor Pippard conducted a series of tests for torsion on redundantly braced frameworks to compare with computed results based on the method developed by R. S. Southwell (29). Stresses for flexure loadings have also been checked. His results are well in agreement with the theoretical values.

Using the same frame, Professor Pippard has made another series of tests to check the similarity between redundantly braced frameworks and solids (30). Different methods of bracings have been employed for the tests of St. Venant's Principle of equivalent loadings. Pippard's results show that when all transverse frames are braced, equivalent loadings could be used as for solids.

Another complete treatment of space frameworks is Professor John Podolsky's "Space Frames, Mechanics of Structures" published in Russian, 1931 (33). In this work, the author devoted his entire volume to all kinds of space structures.

In 1930, Rudolf Bernhard presented an analysis of bridge structures (4). In this paper, the author considered each transverse frame as supported at three elastic corners, and solved the torsion effect by method of work. In the same year, he made a series of tests on a number of bridges with eccentric loadings. His results show that when the bridge is loaded on one track only, the bridge tends to rotate toward the load.

Meanwhile, W. Bergfelder has published another paper dealing with the complete analysis of bridge structures with eccentric loads (3). In this paper, the author analyzed the bridge on the assumption that all joints are pin-connected, and the ends are fully braced with diagonals. His results, however, indicate that when the bridge is loaded on one truss only, the structure will deform in such a way that the loaded transverse frame tends to rotate in a direction opposite to the couple of the eccentric load. This, as noted above, is disapproved by Bernhard's tests.

An early complete treatment of rigid frames in space published in 1926, is due to Alfred Millies, although he has started his work as early as 1921. In his analysis, the author has solved rigid space frames with four and eight legs which are fixed at the bottom. In 1932, the same author has presented a paper for the solution of a cylindrical rigid space frame under vertical and horizontal loads (22). All the analyses of the author are based upon energy considerations.

Professor Benjamin Mayor of Lausanne, Switzerland, has published two papers dealing with the graphical solution of space structures (21); his first paper was presented in 1910, and his second, in 1926. His solution, based on the principle of linear complex, has been presented to the American literature by Professor F. H. Constant (7).

R. v. Mises has used a more practical graphic solution (24), and W. Prager has solved a space structure with the graphic methods of B. Mayor and R. v. Mises (34).

The history of space structures is of very recent date, and the available literature is quite limited.

II. PRELIMINARY INVESTIGATIONS

Methods of stress analysis of bridge structures are based upon the following assumptions:

a. That the bridge is made up of planar structures, and each part is analyzed independently. This is the usual assumption for practical designs. Under this assumption, the immediate deductions are:

The superimposed loads are transmitted from the roadway to the main trusses by simple beam action of the roadway, stringers, and floor beams.

The end reactions of the floor beams are regarded as applied loads on the vertical trusses, which are analyzed as simple structures.

Wind loads are carried by the top and bottom lateral systems. If both diagonals in each panel are assumed to resist both tension and compression, lateral trusses will then be analyzed as redundant trusses.

All transverse frames, including the portals, are considered as rigid frames.

b. That the bridge is a space structure, and all joints are pin-connected in all directions. The simplest space frame starts with four points in space connected by six bars. The degree of redundancy is found from the equation,

$$n = 3m - 6$$

where n is the number of necessary bars for a just stiff structure, and m the number of space joints. Redundant reactions or members are analyzed by strain-energy considerations.

The solution to be presented in this paper is based upon assumptions which are different from both (a) and (b). It differs from (a) in that the bridge is considered as a space structure, and from (b) in that the transverse frames are not pin-connected parallelograms but braced frames which could transmit torsion. A careful consideration of all factors involved would indicate that it is highly desirable to make a critical investigation of the action of portal frames, stringers, top and bottom lateral systems in this section, in order to establish some simple but rational assumptions. Also, the structural properties of the portals established in this section will be used later for the complete solution in Chapter III.

A. PLANAR TRUSSES WITH REDUNDANT MEMBERS AND REACTIONS

1. Stress Computations. The stress analysis of any truss with redundant members or reactions, or both, can be solved by either considering deformations of members, or strain-energy in the structure as a whole. Both approaches would eventually lead to the same fundamental equations:

$$\Sigma \frac{Su_1L}{EA} = 0 \qquad \Sigma \frac{Su_2L}{EA} = 0$$

and placing $S = S' + u_1S_1 + u_2S_2$, these equations become,

$$\frac{\partial W}{\partial S_1} = \Sigma \frac{S'u_1L}{EA} + S_1 \Sigma \frac{u_1^2L}{EA} + S_2 \Sigma \frac{u_1u_2L}{EA} = 0 \tag{1}$$

$$\frac{\partial W}{\partial S_2} = \Sigma \frac{S'u_2L}{EA} + S_1 \Sigma \frac{u_1u_2L}{EA} + S_2 \Sigma \frac{u_2^2L}{EA} = 0 \tag{2}$$

where S = the final stress in the redundant structure.

S' = the stress in the simple structure under external loads, with all redundancies removed.

S_1 = the stress in the redundant member No. 1, when load is applied to the redundant structure. Similarly, S_2 is used for redundant member No. 2.

u_1 = the stress in any member of the structure for a unit tension load in redundant member No. 1, all other redundant members being removed. Also, u_2 is defined in a similar way for redundant member No. 2.

E, A, L, = modulus of elasticity, area of cross-section, and length of any member.

Thus, there will be as many equations as there are redundancies, and stresses could be obtained by solving these equations. Redundant reactions are considered in the same way as redundant members.

2. Displacements. The displacements of any joint in a statically determinate structure may be either determined graphically by drawing Williot diagrams, or algebraically by using the equation,

$$d = \Sigma \frac{SuL}{EA} \qquad (3)$$

where u is the stress in the structure caused by unit load applied at the joint where deflection is required, and in the direction of the desired deflection.

For statically indeterminate structures, the final stresses in all members must first be found from Equations (1) and (2). Deflections of the redundant structure may be obtained either graphically or algebraically. The graphical solution requires a Williot diagram for the structure with all redundant members removed. The algebraic solution is obtained from Equation (3). The u stress is determined for the structure with all redundant members removed; and S, the final stresses in the redundant structure as found from Equations (1) and (2), is applied to the members for which the u stresses were found.

B. THE PORTAL AS A PLANAR FRAME

1. <u>End Floor Beam.</u> The end floor beam of a bridge presents an unusual situation in that it is inclinded at an angle from the plane of the portal frame.

In the bridge which will be used hereafter for stress analysis and as a prototype for model design, the end floor beam consists of the following structural shapes:

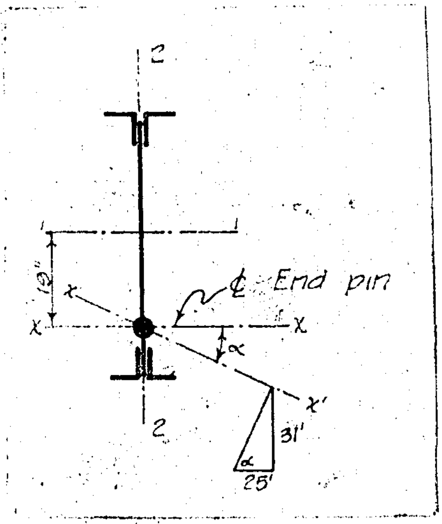


Fig. 1

End Floor Beam

4 L^S 6" x 6" x 9/16"

1 Web $52\frac{1}{2}$" x 3/8"

Area = 45.40 sq.in.

I_{11} = 2046.47 in.^{4}

I_{22} = 181.47 in.^{4}

I_{xx} = 36805.9 in.^{4}

$$I_{x'x'} = \frac{I_x+I_2}{2} + \frac{I_x-I_2}{2}\cos 2\alpha - H \sin 2\alpha$$

$H = \int xyda$ = Product of Inertia with respect to axes xx and 22, and is equal to zero when the section is symmetrical with respect to these axes.

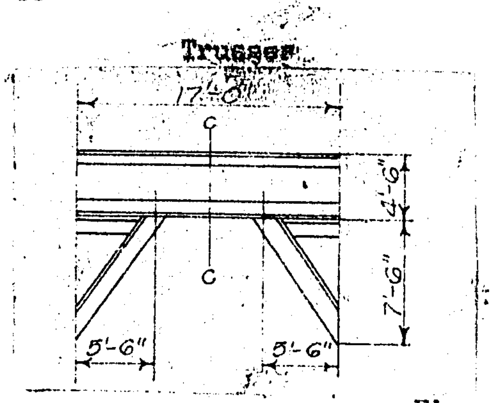
$$I_{x'x'} = \frac{36805.9+181.47}{2} + \frac{36805.9-181.47}{2} \times \cos 2\alpha = 22413.7 \text{ in.}^4$$

$$\frac{4I}{l} = \frac{4 \times 22413.7}{204} = 440 \text{ in.}^3 = \text{Stiffness}$$

2. Head Strut. In through bridges, knee braces are generally used in the portals, and the portal struts are very much stronger at their ends than at the middle sections. "The stiffness at one end of a beam is defined as the moment required to produce unit rotation of this end, when both ends are supported and the other end held fixed." For irregular sections, the easiest way to find the stiffness is to use the method of Column Analogy developed by Professor Hardy Cross (9). Thus, for the strut with dimensions as shown in Fig. (2), if the cross section is assumed uniform through its length and equal to the middle portion, the stiffness is,

$$\frac{4I}{l} = 4 \times \frac{10546.3}{204} = 206.8 \text{ in.}^3$$

However, for this actual section including the knee braces, the stiffness is found equal to 829.01 in.^{3}. For detailed computations, see Appendix B.



Strut: 4 Ls 3" x 3" x 3/8"

1 Web 53½" x 3/8"

Knee 4 Ls 3" x 3" x 3/8"

Braces: 1 Web 3/8" Thick

I_{co} = 10546.3 in.^{4}

Head Strut

Fig. 2

3. Deflection of the Portal Frame. With the stiffness of the individual members in the frame as computed in the preceding sections, the entire portal frame supported at the two lower corners and under either horizontal or vertical loads can be solved in a few ways.

The general method of solution used in the case of the arch with fixed ends can be applied here. The frame is cut at some convenient point, as at the center of the horizontal beam, and deflections at this cut section are then expressed in terms of both the external loads and internal stresses. The relative deflections of the two halves at this section must be equal to zero. There will be as many equations as there are unknown internal stresses, and the solution could be obtained.

Another method is to use the moment distribution method developed by Professor Hardy Cross (9). But the simplest way would be to use the slope deflection method, upon which the following solution is based.

Assuming all clockwise rotations, Θ's, D/L, and rotation produced by outside shears, as positive, the general equation will have the following form:

$$M_{mn} = M_{Fm} - \left(\frac{2I}{I}\right)_{mn} (2\Theta_m + \Theta_n - 3R_{mn}), \text{ for } E = 1$$

Final values of moments will be checked by passing a circle clockwise around any joint. If the first fiber of any member met by this circle is tension, the moment will have a plus sign.

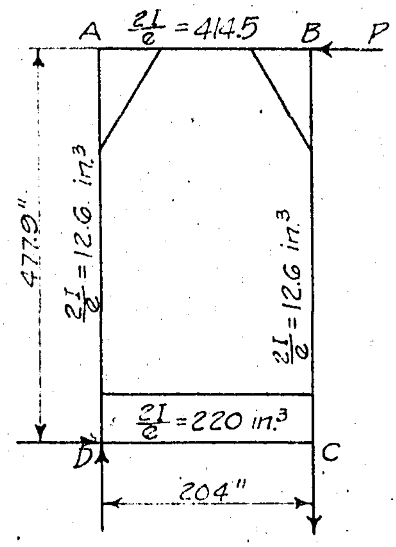


Fig. 3

$I_{AD} = I_{BC} = 3008.6$ in.^{4}

I_{AB}, I_{DC} as found before.

Due to Symmetry,

$$\theta_A = \theta_B$$

$$\theta_D = \theta_C$$

$$R_{AD} = R_{BC} = R$$

Joint A,

$$-1268.7\ \theta_A - 12.6\theta_D + 37.8\ R = 0 \qquad (1)$$

Joint D,

$$-12.6\ \theta_A - 683.2\ \theta_D + 37.8\ R = 0 \qquad (2)$$

$M_{AD} + M_{DA} = -\dfrac{477.9 \times P}{2}$ gives,

$$-37.8\ \theta_A - 37.8\ \theta_D + 75.6\ R + 239\ P = 0 \qquad (3)$$

Solving,

$$\theta_A = -0.097\ P$$

$$\theta_D = -0.187\ P$$

$$R = -3.30\ P$$

$M_{AD} = - 120\ P$

$M_{DA} = - 119\ P$

Actual deflection at B,

$$d_B = \frac{3.30\ P \times 477.9}{E} = \frac{1577\ P}{E} \text{ inches.}$$

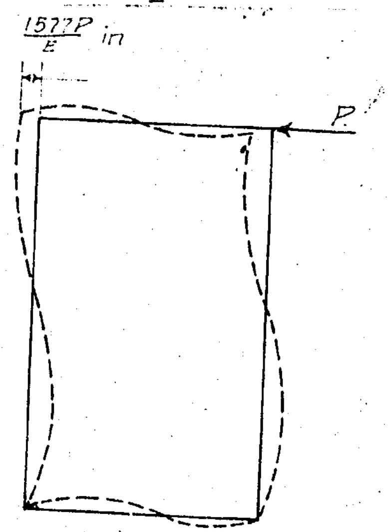


Fig. 4 Portal Deflections.

The moments at the top and bottom of each end post are approximately equal, which shows that the points of inflection of the end posts are very near the middle points of the end posts. That is, the portal frame as a whole has the property of a similar frame with the end posts fixed at the bottom.

C. THE PORTAL AS A SPACE FRAME

1. Method of Investigation. When a member in a space rigid frame is cut at a section, the internal stresses developed at the cut section will have to satisfy six equations:

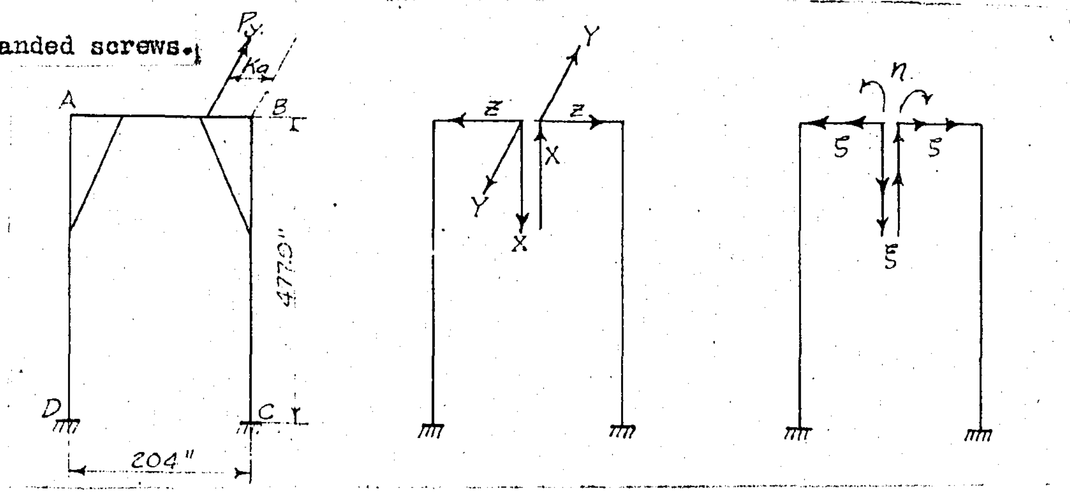
$$\frac{\partial W}{\partial F_x} = 0, \qquad \frac{\partial W}{\partial M_x} = 0,$$

$$\frac{\partial W}{\partial F_y} = 0, \qquad \frac{\partial W}{\partial M_y} = 0,$$

$$\frac{\partial W}{\partial F_z} = 0, \qquad \frac{\partial W}{\partial M_z} = 0.$$

Where, F, M, are forces and moments; and x, y, z, are any three axes perpendicular to each other. Forces consist of shears, and axial stress; and moments consist of bending and torsion. The internal stresses are thus determined from a solution of the above equations.

2. Sign Convention. In the solution of any space frame, the sign convention is important in evaluating the directions of stresses. For standardization, the following sign convention is adopted (23): Directions of forces are shown by single arrow, and those of moments, by double arrow vectors. Double arrow vectors are operated in the sense of right-handed screws.



(a) Force System in Space Frame

(b) Shears & Axial Stress

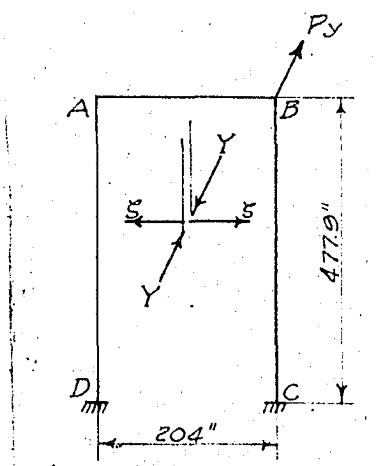
(c) Moments & Torsion

Fig. 5

Space Frame

For a very complicated space rigid frame, members used as beams or struts generally have one sign system, and members used as posts or columns, another. In this case, however, the portal is a very simple frame; internal stresses in the strut as shown in Fig. (5) are assumed as positive.

3. <u>Solution</u>. A solution for the portal as a rigid frame in space will be made in this section. The force P_y is acting as shown in Fig. (6) in a direction perpendicular to the plane of the portal, and applied at the corner B.

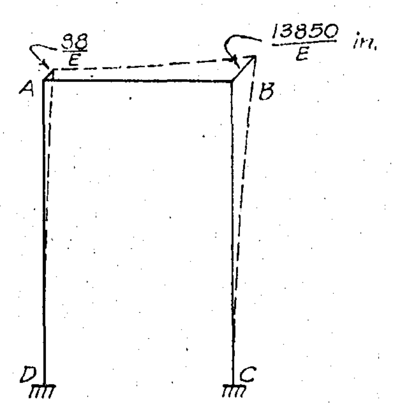


Y = 0.006 P

$\mathcal{S}$ = 0.114P

(a)

Stresses caused by P



(b)

Deflections

Fig. 6

For bending in the plane of the portal:

$$I_{AB} = 10546.3 \text{ in.}^4$$

$$I_{AD},\ I_{BC} = 3008.6 \text{ in.}^4$$

For bending in planes perpendicular to that of the portal:

$$I_{AB} = 17.2 \text{ in.}^4 \text{ (Plane of Lateral Truss)}$$

$$I_{AD},\ I_{BC} = 2611.5 \text{ in.}^4 \text{ (Plane of Vertical Truss)}$$

Polar moments of inertia:

Strut, $J_{AB} = 1.35$ in.^{4}

Post, $J_{AD} = 8.50$ in.^{4}

Assume G, the shearing modulus of elasticity equal to 0.4E.

The polar moment of inertia is assumed equal to $1/3\ \Sigma\, bd^3$ for structural sections.

All structural sections are assumed as made up of small rectangular sections. b, is the longer dimension and d, the shorter dimension of any small rectangular section.

For this loading, only torsion $\mathcal{S}$ and shear Y exist in the strut. Therefore only two equations are required.

$$\frac{\partial W}{\partial Y} = 13931\ P + 2{,}993{,}740\ Y - 87\ \mathcal{S} = 0 \qquad (1)$$

$$\frac{\partial W}{\partial \mathcal{S}} = -\ 43.7\ P - 87\ Y + 378\ \mathcal{S} = 0 \qquad (2)$$

$$Y = -\ 0.006\ P$$

$$\mathcal{S} = +\ 0.114\ P$$

The deflection, (Fig. (6b) of the portal as a space frame is obtained by neglecting the torsion effect of the strut and the end posts. For moments and shears in any other portion of the portal frame, values can be found readily from the known internal stresses in the strut.

D. TOP LATERAL SYSTEM WITH PORTALS AS A SPACE STRUCTURE

The manner in which the portal frames support the top lateral truss, when the load in acting in the plane of this truss, is investigated in this section.

With rigid frames at both ends, the top lateral truss under wind load will be restrained more or less so as to cause slight changes in the axial stresses of all the members. The degree of restraint depends, however, upon the rigidity of the portal frames, and the extreme case will be that when the end posts are assumed fixed at their ends.

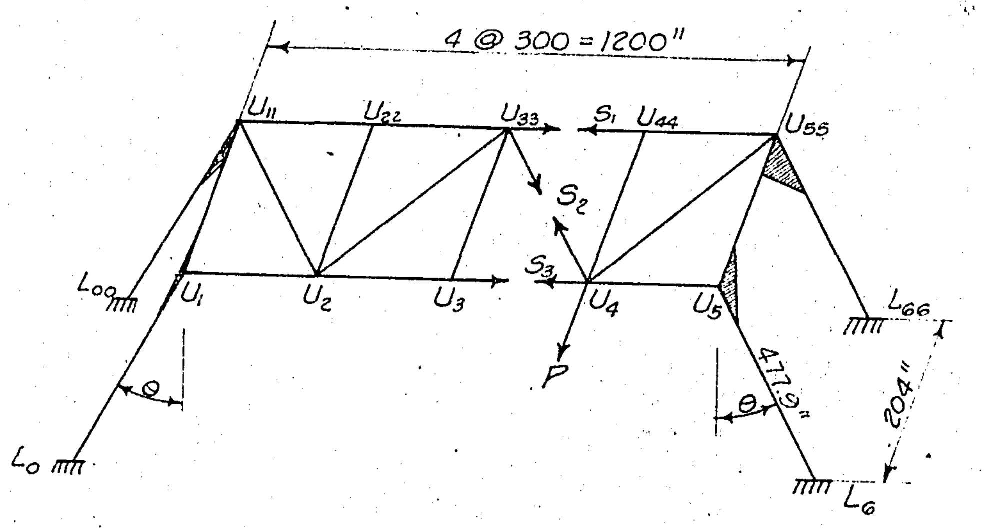


Fig. 7

Top Lateral System as a Space Structure

1. <u>Solution of the Top Lateral System with the End Posts Fixed at the Bottom.</u> A general solution with the end posts fixed at the bottom will be made in this article, since it gives the greatest restraint on the lateral truss. For portals with pin-connected ends, the problem can be solved by a similar analysis. With the properties of the portal frames in space established in sections (B) and (C), the easiest way to obtain a solution is by passing a section through the third panel of the lateral truss. The two halves are then considered as cantilevers.

When all joints of the lateral truss are assumed as pin-connected, and when the portal columns are assumed fixed at the bottom, the portals are stable and can resist loads in the plane of the truss. The three members, thus cut by this section, are considered as redundant members, although the degree of redundancy for the entire system in Fig. (7) is much more than three. For a general solution, the end posts are assumed to make an angle of θ with the verticals.

With the value of L/EA equal to 28.65 for head struts; to 86.79 for top diagonal; to 6.92 for top chords; and with the axial stresses in the end posts and portal struts neglected; an application of the strain-energy principle gives the following three equations:

Then the total deflection along S_1,

$$1 = \frac{S'u_1L}{EA} + S_1\frac{u_1{}^2L}{EA} + S_2\frac{u_1u_2L}{EA} + S_3\frac{u_1u_3L}{EA} + \int\frac{Mmdx}{EI} = 0 \qquad (1)$$

along S_2,

$$2 = \frac{S'u_2L}{EA} + S_1\frac{u_1u_2L}{EA} + S_2\frac{u_2{}^2L}{EA} + S_3\frac{u_1u_3L}{EA} + \int\frac{Mmdx}{EI} = 0 \qquad (2)$$

and

$$\partial_3 = \frac{S'u_3L}{EA} + S_1 \frac{u_1u_3L}{EA} + S_2 \frac{u_2u_3L}{EA} + S_3 \frac{u_3{}^2L}{EA} + \int \frac{Mmdx}{EI} \qquad (3)$$

The last term is the deflection due to portal frames.

Substituting the values obtained before, the final equations will be as follows:

$$E = 1$$

S_1:

$$20244 \cos^2\theta \; P + (27.69 + 27700 \cos^2\theta)S_1$$

$$+ (22.91 + 22920 \cos^2\theta)S_2 + 176 \cos^2\theta S_3 = 0 \qquad (a)$$

S_2:

$$(-171.35 - 884 - 50260 \cos^2\theta)P + (22.91 + 22920 \cos^2\theta)S_1$$

$$+ (423.02 + 993 + 169800 \cos^2\theta)S_2 + 146 \cos^2\theta S_3 = 0 \quad (b)$$

S_3:

$$(-10.18 - 20244 \cos^2\theta)P + 176 \cos^2\theta S_1 + 146 \cos^2\theta S_2$$

$$+ (27.69 + 27700 \cos^2\theta)S_3 = 0 \qquad (c)$$

Substituting in the actual bridge, with $\cos\theta$ equal to 372/477.9, these equations become:

$$16810S_1 + 13910S_2 + 106S_3 + 12266 \; P = 0 \qquad (1)$$

$$13910S_1 + 104298S_2 + 88S_3 - 31507P = 0 \qquad (2)$$

$$106S_1 + 88S_2 + 16810S_3 - 12276P = 0 \qquad (3)$$

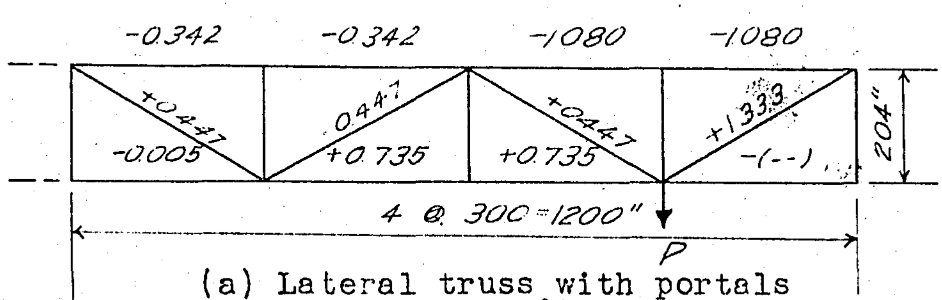
Solving,

$$S_1 = -1.080\ P$$

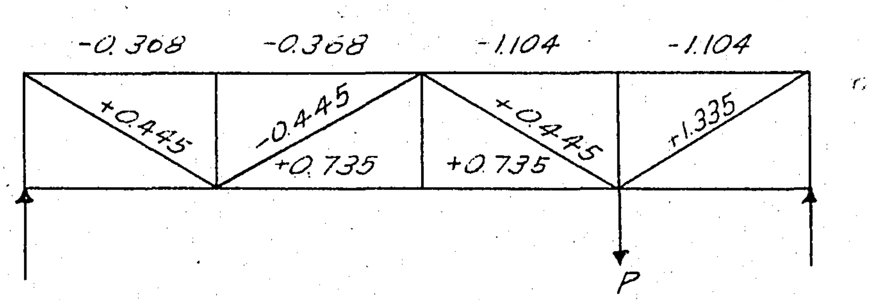
$$S_2 = +0.447\ P$$

$$S_3 = +0.735\ P$$

Stresses in the lateral truss are obtained by substitution.



(a) Lateral truss with portals as a unit.



(b) Lateral truss as simply supported.

Fig. 8.
Stresses in Top
Lateral System.

For the interest of comparison, stresses in the top lateral truss when considered as a simply supported truss are shown in Fig. (8) b. It will be noted that the change in stress is negligibly small. The effect of portal restraint increases as the angle θ increases; and in the extreme case, when θ becomes 90 degrees, the end posts are in the same plane as the lateral truss. In this case, the chord stresses are greatly decreased. For θ less than 60 degrees, the effect of portal restraint can be entirely neglected. A more accurate stress analysis would be to include the axial stresses in the end posts in the general equations.

E. BOTTOM LATERAL INVESTIGATIONS

Investigations in this section are made to determine the effect of stringers on the stresses of other members in the bottom lateral truss.

1. Assumptions. The entire bottom lateral truss, with double diagonal bracing and including all stringers, is a highly redundant structure. Any solution for this system will depend to a large extent upon the way of selecting the basic structure. The basic structure can be chosen in at least two ways. The first way is to assume the whole bottom lateral truss as pin-connected at every joint including the connections between stringers and floor beams. The axial stresses in the stringers will cause bending in floor beams, and cause change in the axial stresses of chord members through reactions. Under

this assumption, there will be twelve redundant stringers, one diagonal in each panel, and one reaction at the top corner, with a total of 19 redundancies.

The second assumption is to regard all the diagonals and the extra reaction as redundant members; and the structure thus left consisting of only floor beams, stringers, and chord members is then considered as a rigid frame. Since the stress in any redundant diagonal can be resolved into components along the direction of the bottom chord and the direction of the floor beam at the joint, the solution of the entire bottom lateral system thus becomes a series of solutions of the rigid frame, for different sets of concentrated loads applied individually at different panel points of the frame.

2. Solution of the Pin-connected Structure. This solution is based upon the first assumption that all joints in the bottom lateral truss are pin-connected; and all stringers, one diagonal in each panel, and one reaction at L_{00}, are regarded as redundancies. Solving in the usual way, the following general equations hold true:

For redundant stringers:

$$\Sigma \frac{SuL}{EA} + \Sigma \int \frac{Mmdx}{EI} = 0 \qquad \text{(A)}$$

For redundant diagonals:

$$\Sigma \frac{SuL}{EA} = 0 \qquad \text{(B)}$$

For redundant reaction, H:

$$\Sigma \frac{SHL}{EA} = 0 \qquad \text{(C)}$$

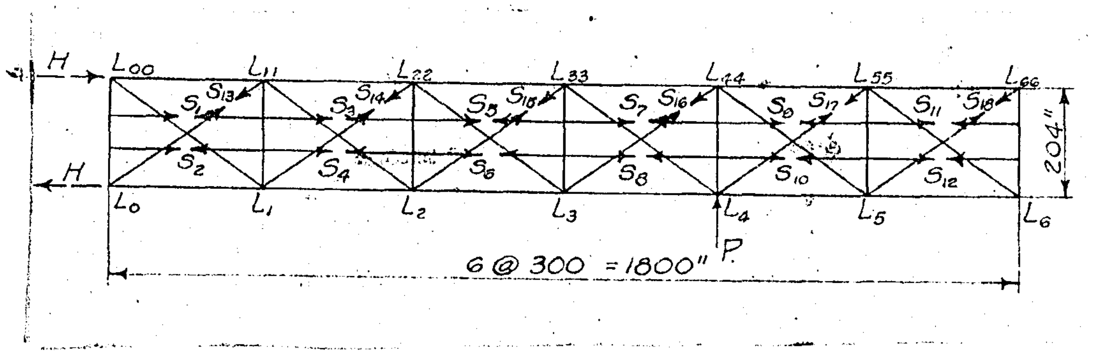


Fig. 9

Bottom Lateral Truss

Put $\quad S = S' + S_1u_1 + S_2u_2 ---- + S_{13}u_{13} ----$

$\qquad + S_{18}u_{18} ---- + Hu_h.$

and $\quad M = M' + S_1m_1 + S_2m_2 ---- + S_{18}m_{18} ----$

$\qquad + Hm_h;$

these general equations have the following forms:

For Stringer No. 1, S_1:

$$S_1\left(\Sigma \frac{u_1^2L}{EA} + \Sigma\int\frac{m_1^2dx}{EI}\right) + S_2\left(\Sigma\frac{u_1u_2L}{EA} + \Sigma\int\frac{m_1m_2dx}{EI}\right)$$

$$+ S_3\Sigma\int\frac{m_1m_3dx}{EI} + S_4\Sigma\int\frac{m_1m_4dx}{EI} + S_{13}\Sigma\frac{u_1u_{13}L}{EA}$$

$$+ H\Sigma\frac{Hu_1L}{EA} + \Sigma\frac{S'u_1L}{EA} + 0 \qquad (1)$$

etc. etc. etc. for stringer No. 12 $\qquad$ (12)

For redundant diagonal, S_{13}:

$$S_1\Sigma\frac{u_1u_{13}L}{EA} + S_2\Sigma\frac{u_2u_{13}L}{EA} + S_{13}\Sigma\frac{u_{13}^2L}{EA}$$

$$+ S_{14} \; \Sigma \frac{u_{13} u_{14} L}{EA} + H \, \Sigma \frac{H u_{13} L}{EA} + \Sigma \frac{S' u_{13} L}{EA} = 0 \qquad (13)$$

etc. etc. etc. for diagonal S_{18} (18)

and for redundant reaction, H:

$$S_1 \; \Sigma \frac{H u_1 L}{EA} + S_2 \; \Sigma \frac{H u_2 L}{EA} + \cdots + S_{13} \; \Sigma \frac{H u_{13} L}{EA}$$

$$+ H \, \Sigma \frac{H^2 L}{EA} + \Sigma \frac{S' H L}{EA} = 0 \qquad (19)$$

Thus there are 19 equations for 19 unknowns; stresses in these redundant members can be found. For this particular bridge, the L/EA and moment of inertia of different members are:

Member	L/EA	I
Bottom chords in panel 1, 2, 5, and 6:	11.33	
in panels 3, 4:	5.96	
Floor beams		
end:	3.61	181.47 in.^{4}
int.:	4.47	286.82 in.^{4}
Diagonals		
panels 1, 6:	74.65	
panels 2-5:	85.97	
Stringers	5.58	

and the solution for this lateral truss under the concentrated load P at L_4 in Fig. (9) gives:

Stringers, in terms of P:

$S_1 = -0.040$	$S_7 = +0.021$
$S_2 = +0.041$	$S_8 = -0.022$
$S_3 = -0.042$	$S_9 = +0.001$
$S_4 = +0.041$	$S_{10} = -0.001$
$S_5 = +0.002$	$S_{11} = +(--)$
$S_6 = -0.003$	$S_{12} = -(--)$

Diagonals, in terms of P:

S_{13}, $L_0 - L_{11} = +0.3890$

S_{14}, $L_1 - L_{22} = +0.3860$

S_{15}, $L_2 - L_{33} = +0.3860$

S_{16}, $L_3 - L_{44} = +0.3812$

S_{17}, $L_4 - L_{55} = -0.5087$

S_{18}, $L_5 - L_{66} = -0.5002$

Redundant reaction,

$H = +0.887$

3. Solution of the System with Stringers Omitted. In this case, it is assumed that the axial stresses in stringers are neglected, and the lateral trusses will have six redundant diagonals and one redundant reaction. This set of equations is obtained from the above

nineteen by omitting the stringer stresses of S_1 and S_{12}. For the same L/EA values of the chord members and diagonals, the results are as follows:

Redundant diagonals:

$L_0-L_{11} = +0.3869P$

$L_1-L_{22} = +0.3835P$

$L_2-L_{33} = +0.3836P$

$L_3-L_{44} = +0.3780P$

$L_4-L_{55} = -0.5108P$

$L_5-L_{66} = -0.5021P$

$H = +0.8670P$

4. Method for Solving the Rigid Frame Structure. This solution is based on the assumption made in (1) that all the twelve diagonals and the reaction H are considered as redundancies; and the basic system with stringers, floor beams, and bottom chords, is then regarded as a rigid frame.

All methods for solving rectangular frames or vierendeel trusses can be well applied here, since the basic frame is in fact an extension of rectangular frames. All redundant diagonal stresses are considered as concentrated loads at all panel points of the frame. Thus, assume a concentrated load P is applied at L_4 in the direction of the floor beam L_4-L_{44}, the Slope Deflection method gives the simplest solution. In this bridge, the moments of inertia are:

Member	I in in.^{4}
Bottom chords in	
panels 1, 2, 5, 6:	4.757
in panels 3, 4:	9.413
End floor beams	
section between stringers:	2.160
section between stringers and bottom chords:	3.025
Int. floor beams	
section between stringers:	3.415
section between stringers and bottom chords:	4.780
Stringers:	0.950

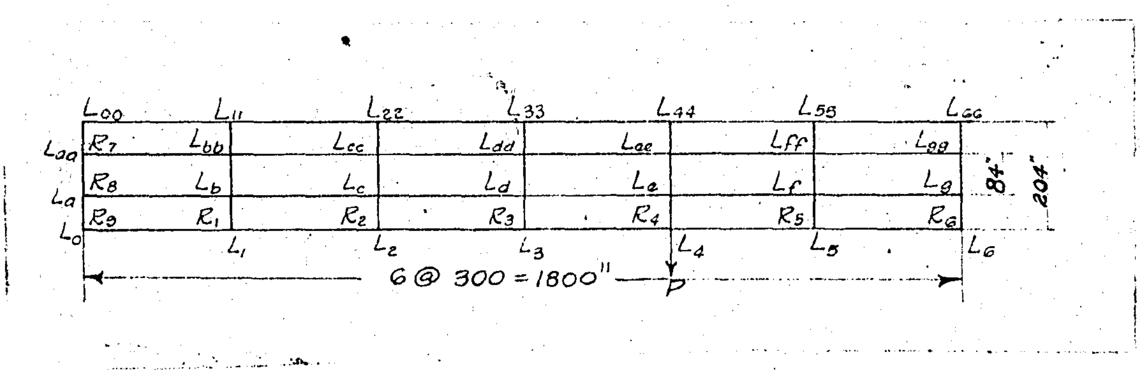


Fig. 10

Bottom Lateral Frame

Due to symmetry, $\theta_0 = \theta_{00}$, $\theta_a = \theta_{aa}$, etc., and $R_7 = R_9$; there will be 14 twist angles and 8 R's as shown in Fig. (10). Using slope deflection method, 14 equations can be obtained by summing all moments about each of the 14 joints equal to zero, and 8 more equations can be obtained from panel shears in vertical and horizontal directions.

Detail solution will not be given here, and results obtained are:

$\theta_a = +0.1721$	$\theta_0 = +1.3435$	$R_1 = +1.95465$
$\theta_b = +0.2409$	$\theta_1 = +1.5201$	$R_2 = +2.03323$
$\theta_c = +0.2087$	$\theta_2 = +1.5145$	$R_3 = +1.69341$
$\theta_d = +0.1202$	$\theta_3 = +1.2840$	$R_4 = +0.82269$
$\theta_e = -0.0826$	$\theta_4 = -0.3554$	$R_5 = -2.76958$
$\theta_f = -0.0936$	$\theta_5 = -2.9691$	$R_6 = -3.73440$
$\theta_g = -0.1391$	$\theta_6 = -2.4421$	$R_8 = +0.07219$
		$R_9 = +0.08287$

By applying concentrated loads at each joint separately, the complete analysis of the entire bottom lateral system can be obtained. Due to symmetry, four sets of equations would be enough for the solution, because the principle of reciprocal deflections can be applied. These four sets of equations are those for the frame loaded with concentrated loads at L_{00}, L_3, L_4, and L_5. Finally, another set of 13 equations is required for the complete analysis; these equations are obtained from the twelve redundant diagonal and the redundant reaction H.

It is at once evident that under this assumption of rigid frame solution, the analysis is extremely tedious. Also, since the axial stresses in the chords, stringers, and floor beams are neglected in the frame solution, the final results are even less accurate.

5. Discussion of Results. From the above analyses, it can be said that the axial stresses in stringers are negligibly small, and their effects on chord stresses can be entirely disregarded.

Also, since the axial stresses in stringers depend upon the lateral stiffness of floor beams, their effects are greatly increased when the floor beams are stiffened laterally. This is the case for highway bridges, where the concrete pavement has stiffened the floor beams to a great extent, and therefore, the axial stresses in chord members may be expected to be considerably reduced.

F. DISPLACEMENT OF SPACE STRUCTURES

The entire bridge structure, with the exception of the two portals, is generally composed of four trusses, and the two planes in which the main trusses lie are perpendicular to the two other parallel planes composing the top and bottom lateral systems. Still in a few other cases, bridges are built up of three trusses so that each transverse frame will form an isosceles or equilateral triangle. But, in any case, the displacements in the plane of any truss can be very easily determined; and in planes other than those in which trusses lie, the displacements can be readily found from vector sums. The following example will clarify the above statements. In the Fig. (11), the frame has the dimensions as indicated, and the three supports A, B, and C are pinned to a fixed plane. With a load of 1000 pounds hung vertically at D, stresses are easily determined as shown in the Fig. (11a).

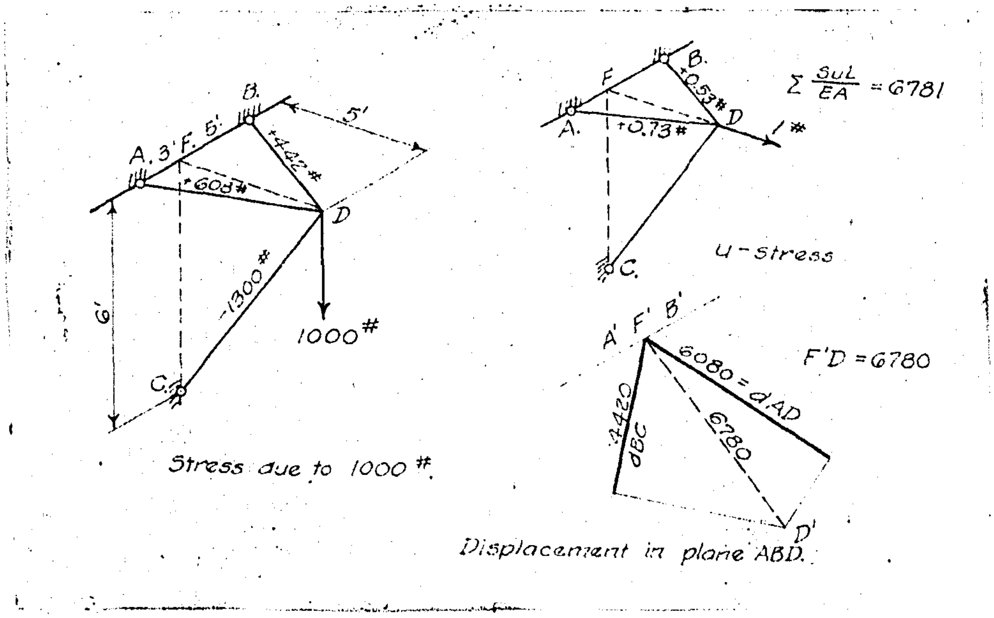


Fig. 11

Assuming the L/A values of all the members are each equal to 10, and the value of E equal to unity, the displacements in the direction of FD, for instance, can be either computed or scaled from a displacement diagram. Thus by computation, a unit load is applied at D in the direction of FD, and stresses u in the frame are as shown in Fig. (11b).

Then, if we call the stresses in the frame due to the 1000 pounds S, the deflection along FD will be simply summation SuL/EA of the whole frame. This computation gives 6781 in a direction away from F, or elongated. By using a displacement diagram, the same result will be obtained as shown in Fig. (11c). It will be noticed that the diagram is based upon the deformations of AD and BD only, since FD is in the plane of ABD which is not affected by the member CD.

In the same way, the vertical deflection at D can be computed or drawn by using the component along FD of the deformations in plane ABD and combining it with that of CD, since CD is in the vertical plane CFD.

III. ANALYSIS OF THE ENTIRE BRIDGE AS A SPACE STRUCTURE

The specific principles and assumptions, upon which is based the solution of the bridge as a space structure, are first discussed.

After the dimensions and structural properties are given for the bridge which is used for this theoretical analysis and as the prototype of a model, a general solution is presented for the rational method of computing the stresses in the bridge considered as a space structure. The special phases of this solution include the computation of stresses resulting from:

(a) Equal loads W_1 placed at the same panel points of both vertical trusses.

(b) A Torque T caused by an eccentric vertical load placed at the panel point of one truss only.

Then a complete stress solution is presented for an eccentric vertical load of 40,000 pounds at joint L_4. The vertical and lateral displacements are also computed for the vertical and lateral trusses and for the transverse frames.

Finally, the secondary stresses are computed in the main members meeting at joint U_4 and produced by the simultaneous bending in each plane of the vertical truss, the top lateral truss and the transverse frame.

A. BASIC PRINCIPLES AND ASSUMPTIONS

Before proceeding with detailed solutions, a few points will be made which will serve to justify the assumptions used hereafter.

1. <u>St. Venant's Principle.</u> General Statement of the Principle.--"An important principle enunciated by St. Venant states that the strains which are produced in an elastic solid, by the application to a small portion of its surface of a system of forces statically equivalent to zero force and zero couple, are of negligible dimensions at distances which are large compared with the linear dimensions of that portion." (30)

When combined with the principle of superposition, an immediate deduction can be made as follows: "Forces applied at one part of an elastic solid will induce stresses which, except in a region close to that part, will depend almost entirely upon their resultant action, and very little upon their distribution." (39)

Applicability of the principle to redundant frames.--St. Venant gave no formal proof of the above principle, which was enunciated with reference to elastic solids, but all experimental evidence since has gone to prove that it is valid, and moreover that the distance from the part of application of the loads is, for practical purposes, a very small one. Based on the considerations of strain-energy, Mr. R. V. Southwell has well demonstrated the truth of the statement (39).

It is of considerable value to interpret a redundant frame from a different angle: A redundant frame is a structure with more bars than necessary to hold external loads or internal strains. If the number of redundancies becomes very large or, as a limit, infinite, the frame will be a solid; and therefore, this principle should be applicable more or less to a redundant frame. The extent of its validity in such cases must depend on the degree of redundancy and the way in which the redundant members are disposed.

Experimental results on the fuselage of airplane structures in the form of tubes braced hexagonally by A. J. S. Pippard and G. H. W. Clifford (30) warrant the following conclusions:

(a) The principle of St. Venant can be applied to the case of a redundant framework.

(b) The operation of the principle is very slow if planes parallel to the loading plane are unbraced, even when the framework exhibits a high degree of redundancy in other planes.

(c) If the plane of loading is braced, the tendency towards the equalization of strains is pronounced, and the distance required for the variation of strains to become negligible is dependent to a marked extent upon the efficiency of the bracing in this and parallel planes (29).

2. Principle of Least Work. The method of least work has played a very important part in the development of the theory of structures, and is very widely used. It has been held that the principle is traceable to the "principle of least action" recognized in the field of mathematical physics. The applicability of this principle in the stress analysis of structures was discovered by Castigliano (1875), and whence it is generally known as Castigliano's second theorem.

This principle briefly stated is: "In a framework which is not initially self-strained, the stresses imposed by a given system of external forces may be found from the conditions for a minimum value of the total strain-energy, if account be taken of the conditions of equilibrium," (39). To put it in a way generally employed in engineering problems, this principle states that the partial differential coefficient of the total strain-energy with respect to the load in a redundant member of a statically indeterminate structure is equal to zero, if that redundant member is of the desired length.

Mr. R. V. Southwell has well demonstrated the physical basis of this principle, and has put it in a more general form so as to be able to apply to frameworks which are initially self-strained (39).

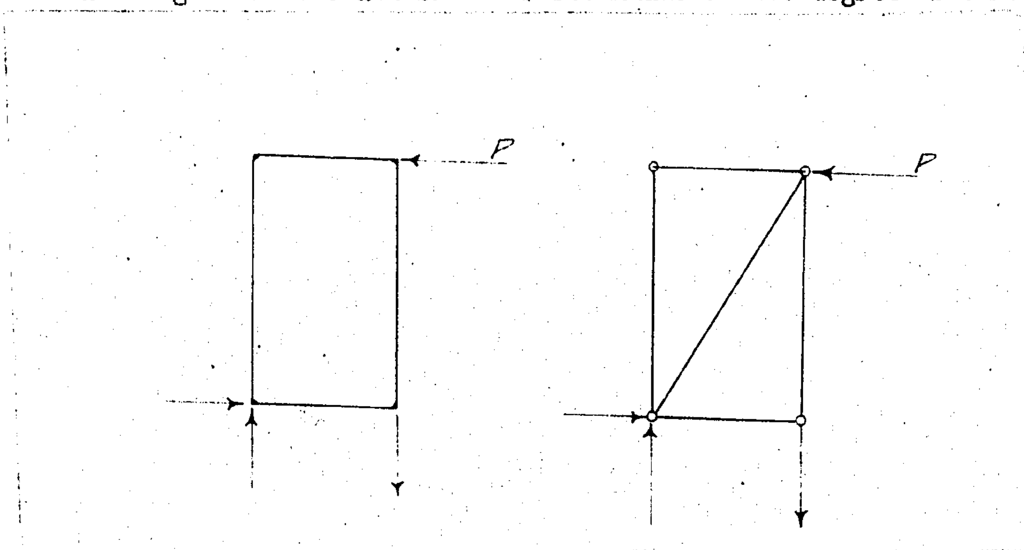
Experimental results on a tubular framework by A. J. S. Pippard and J. F. Baker are in agreement with the theory (31). This result is of interest to engineers, since in aeroplane structures a definite amount of load and stress distribution to the entire frame is very desirable in order to do away with all unnecessary weight.

One point to be noted, however, is that the total differential coefficient of the strain-energy with respect to the load in one of the redundant members would not be equal to zero, if the member is originally not of the exact length. In this case, the coefficient will equal the error in length. This relationship shows that the principle of least work is only a special case of a more general theory.

3. Torsional Resistance of the Transverse Frame. Experiments made by R. Bernhard (3) on a three truss, double track, railway bridge and other tests have shown that, with locomotives on one track only, the

bridge will rotate slightly in the direction of the torque created by the eccentric load. Also, ordinary computations on planar structures usually indicate that the separate transverse frames of a through bridge act as rigid frames. That is, they have never been regarded as parallelograms pinned at the four corners.

Again, in the rectangular frame shown in Fig. (12), if the four corners A, B, C, D, are pinned, a total number of five members will make it determinate. Actually, counting each stiff corner as one member, there are eight members and so it is redundant to the degree of three.



(a) Rigid Frame (b) Determinate Structure

Fig. 12

Analytically, it can be regarded as a cross frame of six members and two members continuous over any two of the four corners as shown in Fig. (13), and with such imaginary elastic properties as to make the entire structure behave the same as the rigid rectangular frame. Experiments made on aeroplane structures, however, have shown that the stresses in the transverse bracing wires are negligibly small and have little effect on other members. Therefore, refining details such as imaginary diagonal members in rectangular frames could be omitted.

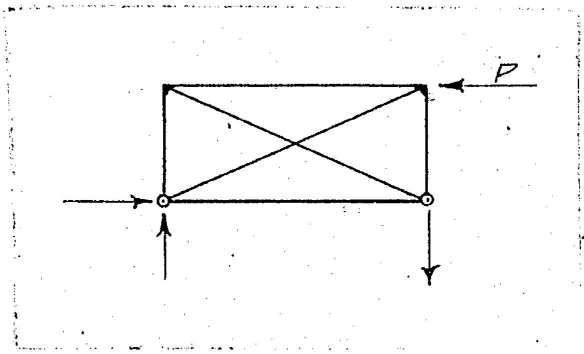


Fig. 13

Frame Equivalent to Rectangular Frame

4. Fundamental Assumptions. The solution of the entire structure is based upon the following assumptions:

a. All four trusses composing the bridge are pin-connected in their own planes.

b. Transverse frames are redundantly braced cross-frames in the form of rigid corners as far as equivalent loading systems are concerned. Fictitious members to replace the rigid corners are neglected.

c. Axial stresses in stringers are neglected.

d. Effect of portals on top lateral system is ignored; the lateral system is regarded as simply supported.

e. Deformations and deflections due to secondary stresses are neglected.

f. Principle of St. Venant applies.

g. Principle of Superposition applies.

h. Principle of Least Work applies.

i. All members are able to resist both tension and compression.

j. Materials, being elastic, obey Hooke's Law.

B. SOLUTION

1. General Statement of the Problem. In the following articles, a complete solution for a 150-foot single track through railway bridge will be made as a space structure. The displacements as well as stresses will be computed. Secondary stresses will be determined for two adjacent trusses and a transverse frame so that the bending of certain members in space could be studied. This bridge has been used for various studies by former investigators. Dr. Von Abo, using many available methods, has made detailed computations of secondary stresses for one of its main trusses. The principal dimensions are as follows:

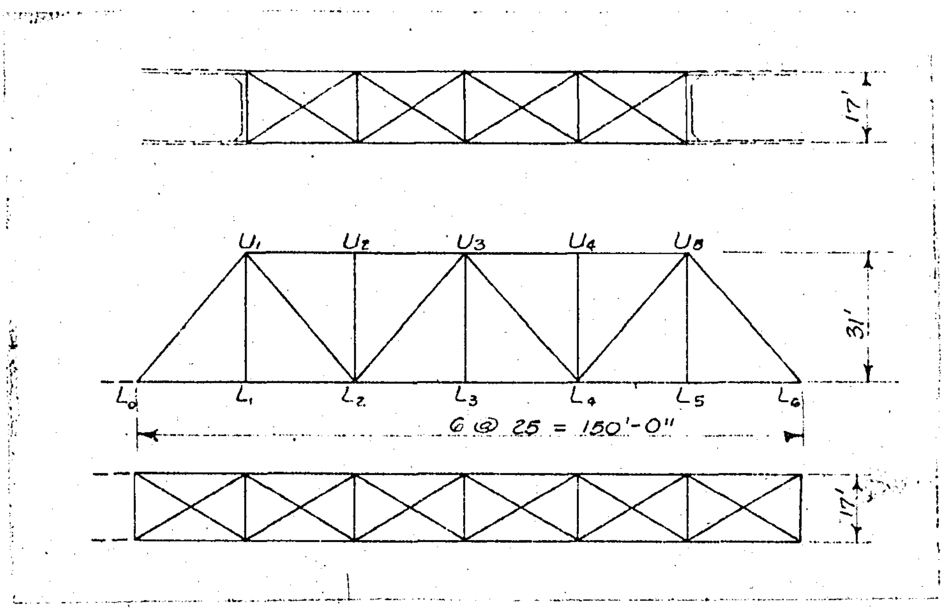


Fig. 14

ESSENTIAL DESIGN DATA

	Length L in in.	Area A in sq.in.	L/A
a. Main truss			
Top chords	300	43.33	6.92
Bottom chords			
2(end panels)	300	26.48	11.33
(2 mid-panels)	300	50.36	5.96
Diagonals			
(end posts)	477.9	49.45	9.66
(panels 2 & 5)	477.9	29.42	16.24
(panels 3 & 4)	477.9	32.36	14.77
Verticals			
(2 end ones & middle)	372	24.35	15.27
(U_2-L_2, U_4-L_4)	372	12.20	30.49
b. Top laterals			
Portal struts	204	28.50	7.16
Head struts	204	7.12	28.65
Lateral diagonals	362.8	4.18	86.79

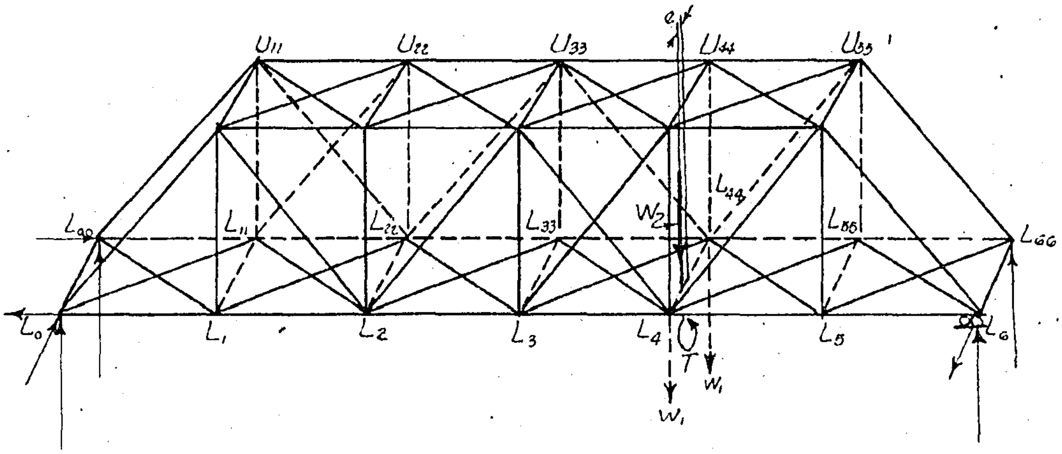
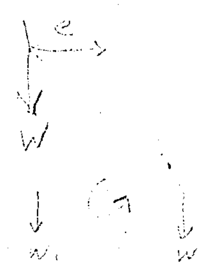


Fig. 15.

Bridge as a Space Structure
under eccentric load.



ESSENTIAL DESIGN DATA (CONT.)

	Length L in in.	Area A in sq.in.	L/A
c. Bottom laterals			
Floor beams			
(2 end ones)	204	45.40	4.49
(int. ones)	204	56.47	3.61
Diagonals			
(2 end panels)	362.8	4.86	74.65
(int. panels)	362.8	4.22	85.97

d. Portal frame

Moments of inertia in the plane of portals

Portal strut	42279 $in.^4$
End posts	3008.6 $in.^4$
End floor beams	22413 $in.^4$

For detailed Section of Members, see prototype, model sections.

2. Methods of Analysis. In Fig. (15), a simple bridge considered as a space structure is acted on by a vertical load W which is placed off center with eccentricity e. With the assumptions made, the load could be divided up into two symmetrical loads w_1 identically placed on the two trusses, and a statically equivalent torque T equal to W e acting as shown.

Stresses due to the two identical loads w_1 can be easily computed as for simple structures except that the lateral bracing will share a part of the chord stresses because of the elastic property of materials.

Effect of the torque will receive special consideration. Undoubtedly, there can be several ways to distribute the torque among members. In Fig. (16), the torque is divided up into two couples acting on the four trusses. Torque T equals Ph + Qb, and the relative value between P and Q should be such that the total amount of energy stored in the entire structure must be a minimum.

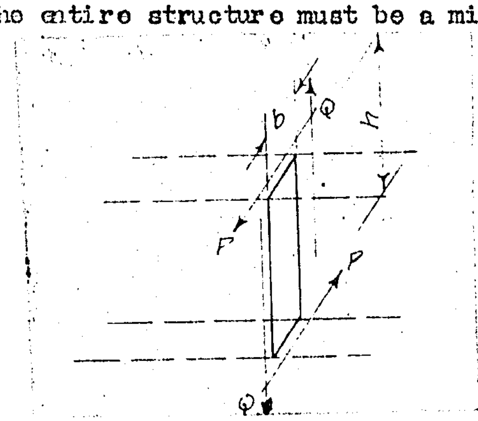


Fig. 16

Or, converting Q in terms of T and P,

$$Q = \frac{T - Ph}{b}$$

Let W = total Strain-energy in the Bridge,

Then

$$\frac{\partial W}{\partial P} = 0.$$

For axial Stresses,

$$\frac{\partial W}{\partial P} = \sum \frac{SL}{EA} \cdot \frac{\partial S}{\partial P} ;$$

and for bending moments,

$$\frac{\partial W}{\partial P} = \sum \int \frac{Mdx}{EI} \cdot \frac{\partial M}{\partial P}$$

Stresses in the verticals and diagonals of the main trusses are caused only by Q, and those in the chords and end posts will result from both P and Q and the effect of lateral bracing. This effect is caused by the distortion of members. With the assumptions made that the top lateral is simply supported by the portals, which is very near to the actual conditions, moments in the portals are to be considered in addition to their axial stresses.

3. Stresses Caused by Symmetrical Loads w_1. For convenience, the computation of stresses caused by the two identical loads w_1 at L_4, L_{44} are taken up in the order of vertical trusses, top lateral, and bottom lateral systems.

(a) Stresses in the Vertical Trusses (one concentrated load at identical joints of each truss).--

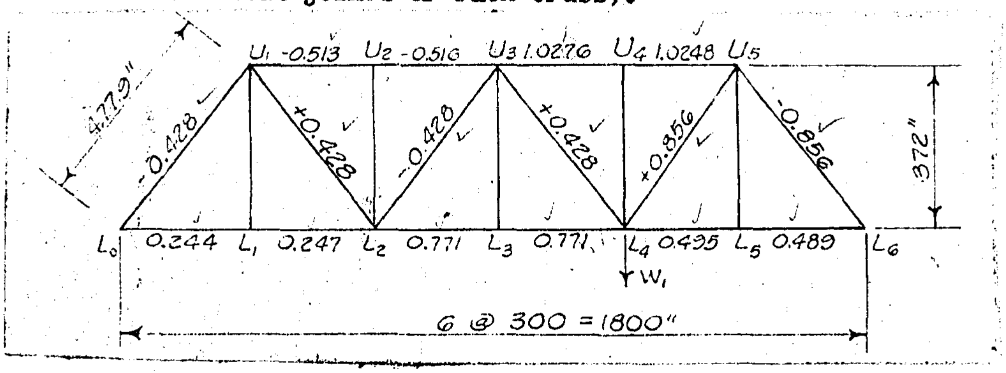


Fig. 17

Because of the symmetry and identity of the loads on the two trusses, stresses are the same in each. Since there are no other members except the main diagonals to resist the vertical shear, stresses in these diagonals and end posts would be the same as for simple structures. They are shown on those members in Fig. (17).

Stresses in the chord members were obtained by the analyses described in the following Sections (b) and (c).

(b) Top Lateral System.--Assume that a section of the bridge is cut just below the top lateral system; forces acting on this system will be only the horizontal components of stresses in the diagonals of the vertical trusses. This force system is shown in Fig. (18). The lateral truss is then solved in the usual way as a redundant structure.

Consider the four diagonals U_1-U_{22}, U_2-U_{33}, U_3-U_{44}, and U_4-U_{55} as redundant members, four equations can be set up from energy considerations.

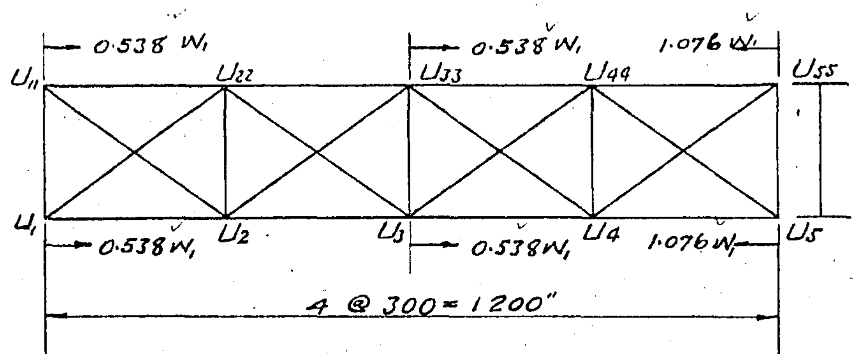
$$\frac{SL}{EA} \cdot \frac{\partial S}{\partial S_1} = \sum \frac{S'u_1L}{EA} + S_1 \sum \frac{u_1{}^2L}{EA} + S_2 \sum \frac{u_1u_2L}{EA} = 0 \qquad (1)$$

etc. etc. etc. (4)

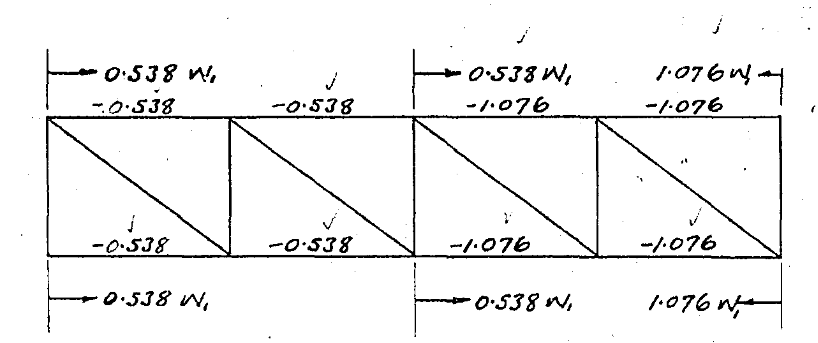
From the given dimensions of individual members, the differential coefficients of the four energy equations can be best found by considering each panel separately in order to reduce space.

Panel	$\sum \frac{u^2L}{EA}$	$\sum \frac{S'uL}{EA}$
1	194.357	6.160
2	201.146	6.160
3	201.146	12.320
4	194.357	12.320

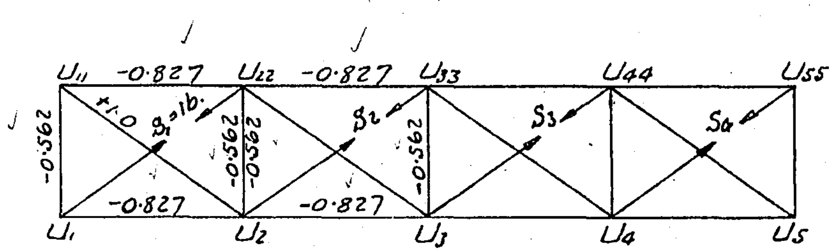
Also for any two consecutive panels, summation of u_1u_2L/EA etc., will concern only the common head strut between them; and, in this case, is



(a) Force Syetem
in redundant truss.



(b) Stress S' in
simple structure.



(c) Stress u in
simple structure.

Fig. 18.
Top Lateral System.

equal to 9.050 for any two consecutive panels. Substituting these coefficients, the required equations are:

$$194.357S_1 + 9.050S_2 + 6.160W_1 = 0 \qquad (1)$$

$$9.050S_1 + 201.146S_2 + 9.050S_3 + 6.160W_1 = 0 \qquad (2)$$

$$9.050S_2 + 201.146S_3 + 9.050S_4 + 12.320W_1 = 0 \qquad (3)$$

$$9.050S_3 + 194.357S_4 + 12.320W_1 = 0 \qquad (4)$$

Solving, the following values are obtained:

$$S_1 = -0.03045W_1$$

$$S_2 = -0.02667W_1$$

$$S_3 = -0.05732W_1$$

$$S_4 = -0.06066W_1$$

Substitute back into $S = S' + S_1u_1 + S_2u_2 + \dots$

the final stresses will be as shown in Fig. (19).

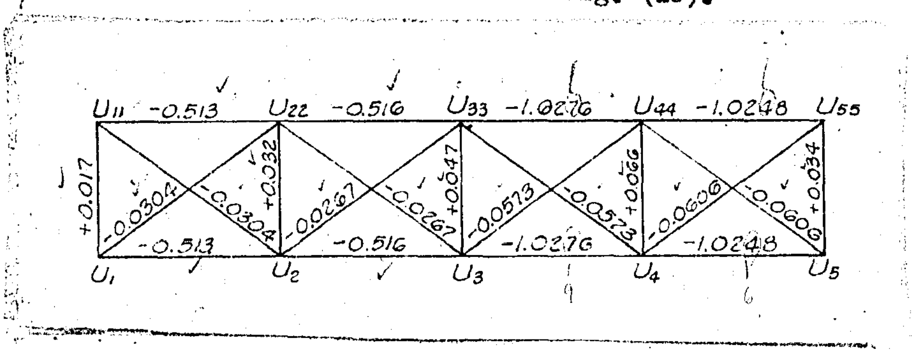
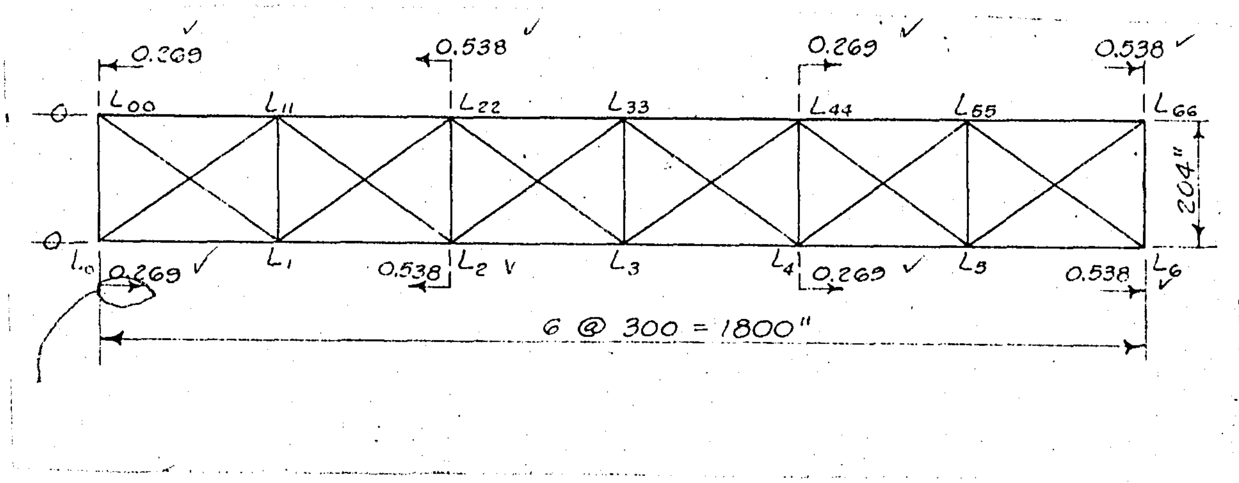
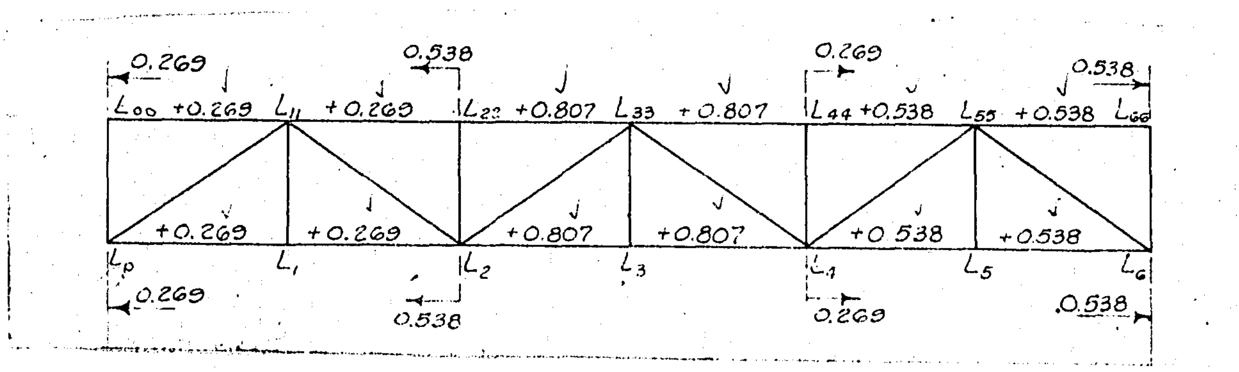


Fig. 19 Final Stresses in Top Lateral When One Concentrated load W_1 placed at L_{44} Vertically.

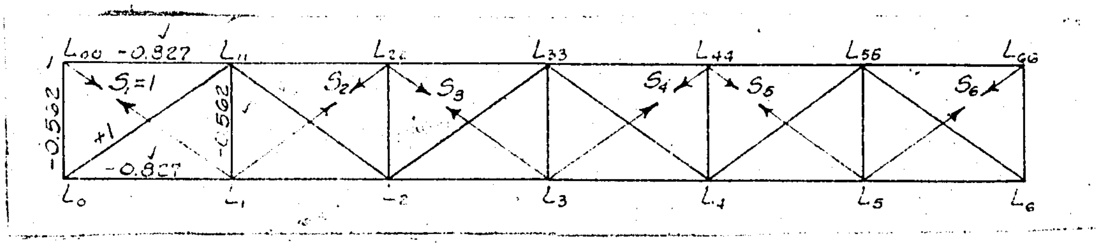
(c) Stresses in the Bottom Lateral System.--By cutting a section through the entire structure just above the floor system, external forces acting on the bottom laterals are those due to the horizontal components of the diagonals in the main trusses only. They are shown as follows:



(a) Forces Acting on Redundant
Bottom Lateral System



(b) S' Stresses in Terms of w_1
in the Simple Structure



(c) u Stresses in Simple Structures

Fig. 20

The stresses in the diagonals are solved as for the top laterals. For symmetrical loads on the two vertical trusses the bottom chords are equally strained, therefore, no horizontal reactions will result.

The necessary differential coefficients in the equations for the six redundant diagonals are listed as follows:

Panel	$\sum \frac{u^2L}{EA}$	$\sum \frac{S'uL}{EA}$
1	167.358	-5.040
2	189.720	-5.040
3	182.374	-7.954
4	182.374	-7.954
5	189.720	-10.080
6	167.358	-10.080

and u_1u_2L/EA, u_2u_3L/EA, etc., each term is equal to 1.141. The six equations are set up as:

$$167.358S_1 + 1.141S_2 - 5.040w_1 = 0 \qquad (1)$$

$$1.141S_1 + 189.720S_2 + 1.141S_3 - 5.040w_1 = 0 \qquad (2)$$

$$1.141S_2 + 182.374S_3 + 1.141S_4 - 7.954w_1 = 0 \qquad (3)$$

$$1.141S_3 + 182.374S_4 + 1.141S_5 - 7.954w_1 = 0 \qquad (4)$$

$$1.141S_4 + 189.720S_5 + 1.141S_6 - 10.080w_1 = 0 \qquad (5)$$

$$1.141S_5 + 167.358S_6 - 10.080w_1 = 0 \qquad (6)$$

On solving, the values of S_1, S_2, ----, are as follows:

$$S_1 = +0.02980w_1$$

$$S_2 = +0.02613w_1$$

$$S_3 = +0.04318w_1$$

$$S_4 = +0.04302w_1$$

$$S_5 = +0.05251w_1$$

$$S_6 = +0.05987w_1$$

Final stresses in the bottom laterals due to symmetrical loads: Chord stresses will be reduced by 0.827 times the load of the redundant diagonal in that panel; floor beams will be stressed to the extent of 0.562 times the 2 adjacent diagonal stresses.

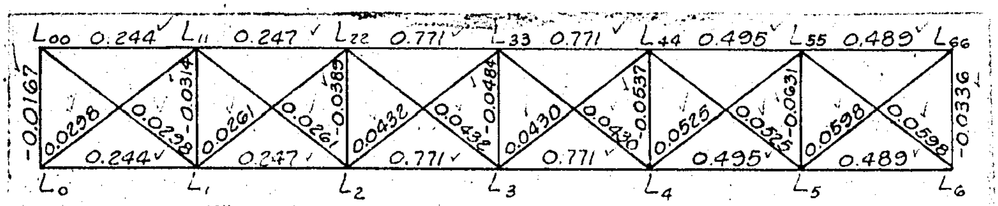


Fig. 21
Stresses in terms of w_1, which is placed vertically at L_4, L_{44} in the main trusses.

TABLE II. STRESSES DUE TO w_1 AT L_4, L_{44}

	Member	Stresses in terms of w_1
Top Chords	U_1-U_2	- 0.513
	U_2-U_3	- 0.516
	U_3-U_4	- 1.0276
	U_4-U_5	- 1.0248
Bottom Chords	L_0-L_1	+ 0.244
	L_1-L_2	+ 0.247
	L_2-L_3	+ 0.771
	L_3-L_4	+ 0.771
	L_4-L_5	+ 0.495
	L_5-L_6	+ 0.489
Main Truss Diagonals	U_1-L_0	- 0.428
	U_1-L_2	+ 0.428
	U_3-L_2	- 0.428
	U_3-L_4	+ 0.428
	U_5-L_4	+ 0.856
	U_5-L_6	- 0.856

	Member	Stresses in terms of w_1
Main Truss Verticals	U_1-L_1	-----
	U_2-L_2	-----
	U_3-L_3	-----
	U_4-L_4	-----
	U_5-L_5	-----
Head Struts	$U_{11}-U_1$	+ 0.017
	$U_{22}-U_2$	+ 0.032
	$U_{33}-U_3$	+ 0.047
	$U_{44}-U_4$	+ 0.066
	$U_{55}-U_5$	+ 0.034
Floor Beams	$L_{00}-L_0$	- 0.0167
	$L_{11}-L_1$	- 0.0314
	$L_{22}-L_2$	- 0.0389
	$L_{33}-L_3$	- 0.0484
	$L_{44}-L_4$	- 0.0537
	$L_{55}-L_5$	- 0.0631
	$L_{66}-L_6$	- 0.0336

	Member	Stresses in terms of w_1
Top Diagonals	U_1-U_{22}	- 0.0304
	$U_{11}-U_2$	- 0.0304
	U_2-U_{33}	- 0.0267
	$U_{22}-U_3$	- 0.0267
	U_3-U_{44}	- 0.0573
	$U_{33}-U_4$	- 0.0573
	U_4-U_{55}	- 0.0606
	$U_{44}-U_5$	- 0.0606
Bottom Diagonals	L_0-L_{11}	+ 0.0298
	$L_{00}-L_1$	+ 0.0298
	L_1-L_{22}	+ 0.0261
	$L_{11}-L_2$	+ 0.0261
	L_2-L_{33}	+ 0.0432
	$L_{22}-L_3$	+ 0.0432
	L_3-L_{44}	+ 0.0430
	$L_{33}-L_4$	+ 0.0430
	L_4-L_{55}	+ 0.0525
	$L_{44}-L_5$	+ 0.0525
	L_5-L_{66}	+ 0.0598
	$L_{55}-L_6$	+ 0.0598

Stresses in truss TT are the same as in trussT

It is interesting to note that, due to symmetrical loads on each truss, the top chord stress at the left end has been reduced from $0.538w_1$ to $0.513w_1$ on account of lateral bracing; the bottom chord stress at left has been reduced from $0.269w_1$ to $0.244w_1$. Percentage of reduction for the top chord is about 4.6%, and for the bottom chord 9.15%. Thus in the limiting case, if the lateral bracing is made of non-elastic and high stress materials, stresses in the chord members will become zero, or a reduction of 100 per cent may be expected.

4 (a) Vertical Trusses.--With Q acting on the truss, stresses in the diagonals are computed as for a simple structure, since P has no effect on them. Reaction stresses on end posts received from top lateral must be considered. For chord stresses, see the lateral systems.

4 (b) Top Lateral Truss.--When the couple Q acts alone, top chords in truss TT would be stretched to the same amount as those would be shortened in truss T, and consequently the top diagonals will not be stressed by Q. Considering the top lateral as a simply supported truss, chord stresses will be those due to both P and Q, while diagonal stresses will be computed for P only.

Again solving by the theory of strain-energy, values of the redundant diagonals can be readily found:

4. Stresses Caused by Torque T:

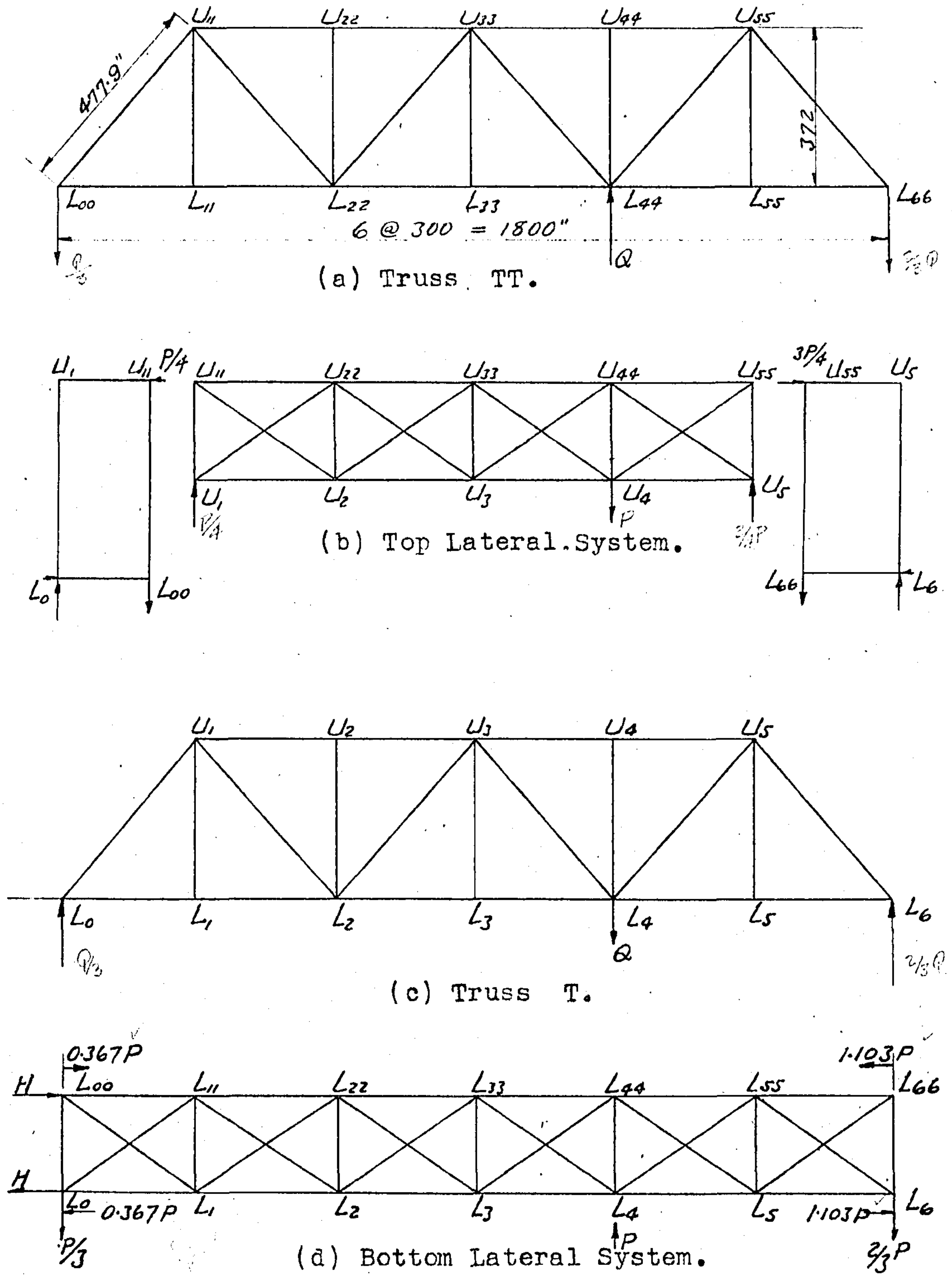


Fig. 22. Force System due to Torque.

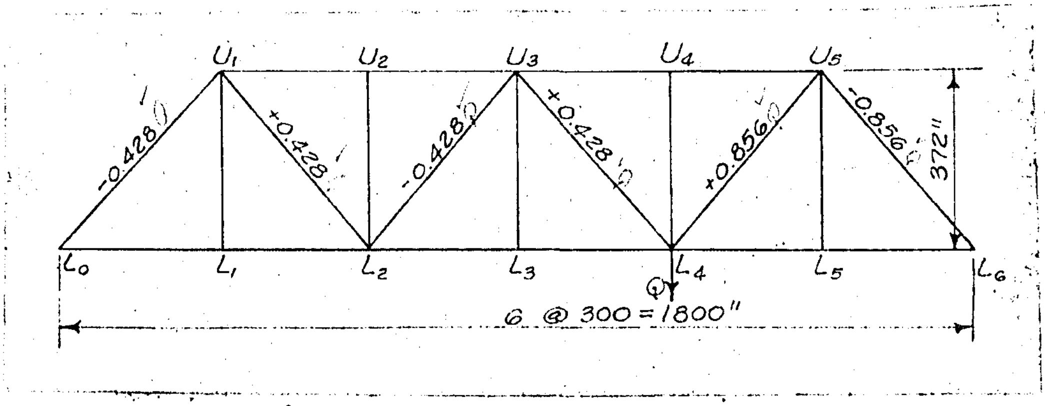


Fig. 23

Assuming point of inflection half way between L_0 and U_1

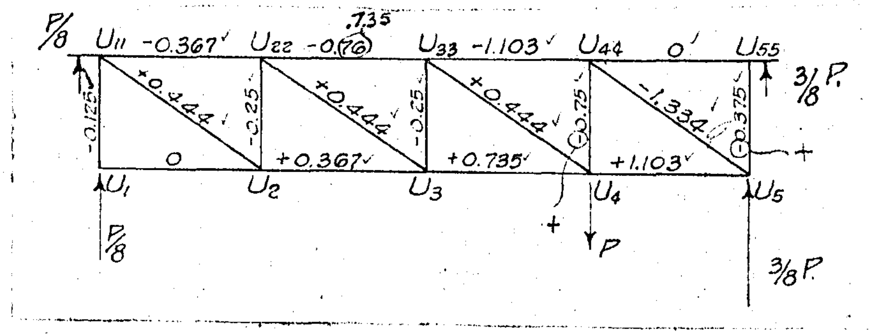
P stress at left end post $(\frac{P}{4} \times \frac{477.9}{204}) \times \frac{1}{2} = 0.293$ P

P " " right " " $3 \times 0.293 = 0.879$ P

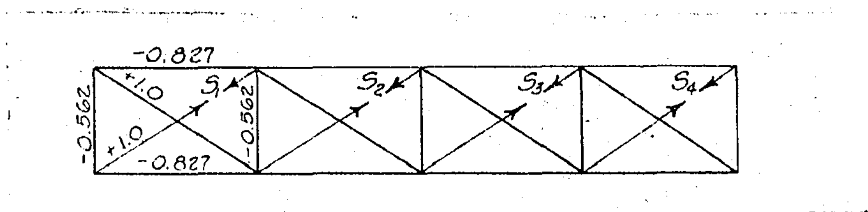
TABLE III. STRESSES AND ENERGY COEFFICIENTS IN VERTICAL TRUSS

Mem.	$\frac{L}{A}$	S		S		$\frac{SL}{EA} \cdot \frac{\partial S}{\partial P}$	
		P	Q	T	P	T	P
U_1-L_0	9.66	-0.293	-0.428	-0.02516	0.488	-0.1186	2.300
U_1-L_2	16.24	----	+0.428	+0.02516	-0.781	-0.3192	9.908
U_3-L_2	14.77	----	-0.428	-0.02516	+0.781	-0.2902	9.008
U_3-L_4	14.77	----	+0.428	+0.02516	-0.781	-0.2902	9.008
U_5-L_4	16.24	----	+0.856	+0.0503	-1.561	-1.2700	39.582
U_5-L_6	9.66	-0.879	-0.856	-0.0503	0.682	-0.3310	4.493
Verts. No Stress							
						Σ -2.619	74.299

$\Sigma \frac{SL}{EA} \cdot \frac{\partial S}{\partial P}$ for truss TT same



(a) Simple Stresses



(b) u Stresses

Fig. 24

Panel	$\Sigma \frac{u^2L}{A}$	$\Sigma \frac{u_1u_2L}{A}$,	$\Sigma \frac{S'uL}{A}$
1	194.357	$\Sigma \frac{u_2u_3L}{A}$,	45.165
2	201.146	etc.	48.693
3	201.146	"	32.593
4	194.357	+9.050	-135.683

Setting up equations:

$$194.357S_1 + 9.050S_2 + 45.165\ P = 0 \qquad (1)$$

$$9.050S_1 + 201.146S_2 + 48.693\ P = 0 \qquad (2)$$

$$9.050S_2 + 201.146S_3 + 32.593\ P = 0 \qquad (3)$$

$$9.050S_3 + 194.357S_4 - 135.683\ P = 0 \qquad (4)$$

Solving,

$$S_1 = -0.2219\ P$$

$$S_2 = -0.2238\ P$$

$$S_3 = -0.1837\ P$$

$$S_4 = +0.7066\ P$$

and $\quad Q = 0.0588\ T - 1.824\ P \quad$ (foot pounds)

The reactions due to force P acting on the top lateral truss produce definite stresses in the end posts of the portals. The horizontal components of these stresses may later be considered as applied loads on the bottom lateral truss.

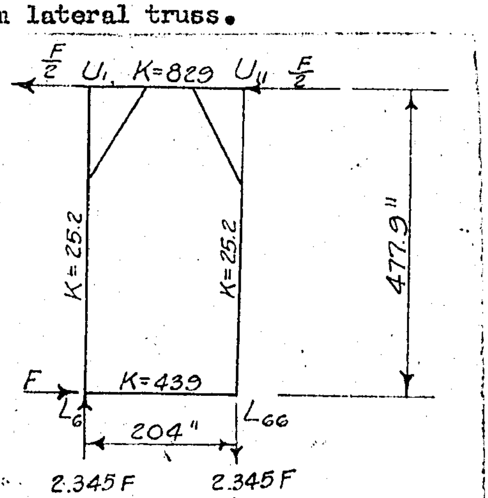


Fig. 25

TABLE IV. STRESSES AND ENERGY COEFFICIENTS IN TOP LATERALS

(1)		(2)	(3)		(4)		(5)	
			$S = A_{\circ}Q + B_{\circ}P + C_{\circ}S_{1,2}$		S		$\frac{SL}{EA} \cdot \frac{\partial S}{\partial P}$	
Member		$\frac{L}{A}$	Q	P	T	P	T	P
Top Chords	$U_{11}-U_{22}$	6.92	0.538	-0.181	+0.0316	-1.164	-0.254	+ 9.348
	$U_{22}-U_{33}$	6.92	0.538	-0.550	+0.0316	-1.531	-0.335	+16.227
	$U_{33}-U_{44}$	6.92	1.075	-0.952	+0.0632	-2.912	-1.274	+58.745
	$U_{44}-U_{55}$	6.92	1.075	-0.577	+0.0632	-2.545	-1.111	+44.800
	$U_{1}-U_{2}$	6.92	-0.538	+0.185	-0.0316	+1.164	-0.255	+ 9.412
	$U_{2}-U_{3}$	6.92	-0.538	+0.552	-0.0316	+1.533	-0.335	+16.269
	$U_{3}-U_{4}$	6.92	-1.075	+0.886	-0.0632	+2.848	-1.246	+56.153
	$U_{4}-U_{5}$	6.92	-1.075	+0.526	-0.0632	+2.480	-1.088	+42.785
Struts	$U_{11}-U_{1}$	7.16	----	----	----	----	----	----
	$U_{22}-U_{2}$	28.65	----	+0.001	----	+0.001	----	----
	$U_{33}-U_{3}$	28.65	----	-0.021	----	-0.021	----	+0.013
	$U_{44}-U_{4}$	28.65	----	+0.456	----	+0.456	----	+5.960
	$U_{55}-U_{5}$	7.16	----	-0.022	----	-0.022	----	+0.024
Diagonals	$U_{11}-U_{2}$	86.79	----	+0.222	----	0.222	----	+4.201
	$U_{1}-U_{22}$	86.79	----	-0.222	----	-0.222	----	+4.355
	$U_{22}-U_{3}$	86.79	----	+0.220	----	0.220	----	+4.239
	$U_{2}-U_{33}$	86.79	----	-0.224	----	-0.224	----	+4.316
	$U_{33}-U_{4}$	86.79	----	+0.261	----	0.261	----	+5.912
	$U_{3}-U_{44}$	86.79	----	-0.184	----	-0.184	----	+2.907
	$U_{44}-U_{5}$	86.79	----	-0.627	----	-0.627	----	+34.100
	$U_{4}-U_{55}$	86.79	----	0.707	----	0.707	----	+43.410
							$\Sigma = -5.898$	$\Sigma = 363.166$

$\int \frac{Mds}{EI} \cdot \frac{\partial M}{\partial F}$ = deflection at U_1 due to $F = \frac{1577}{E}$

For left portal $= \frac{1577}{E} \times (\frac{1}{4})^2 P = \frac{98.5}{E}$

For right portal $= \frac{1577}{E} \times (\frac{3}{4})^2 P = \frac{887.0}{E}$

The total differentiation of all moments in the two portals with respect to P in top lateral gives $\frac{1}{E}(98.5 + 887.0) = 985.5$

Considering the point of inflection of the end post at its center, axial stresses in the end posts from portal action will be $(\frac{P}{4} \times \frac{477.9}{204} \times \frac{1}{2}) = 0.293\ P$ for the left, and 3×0.293 or $0.879\ P$ for the right portal.

4 (c) **Bottom Lateral Truss.**--Again as in top laterals, bottom diagonals would not be stressed any due to Q if not for the redundant reaction at L_{00} which holds that point fast. To hold that point rigid, stresses will be induced in the diagonals. Total forces acting on the bottom lateral system are those due to P, Q, and the reactions from portals which should be taken care of on account of roller ends.

Remove all the redundant diagonals and the extra reaction H, the simple structure will be acted on as shown in Fig. (26).

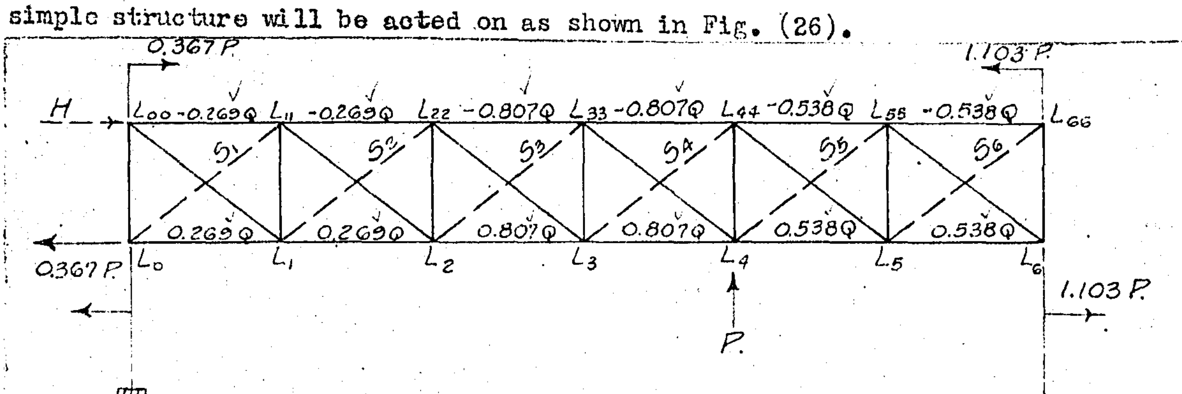


Fig. 26

The dotted members and reaction H are taken as redundant members. Forces at joints L_{00}, L_{66}, L_0, and L_6 are axial stresses from the portals when P is acting on top lateral system.

Converting the values of Q in terms of P and T, Q being equal to (0.0583 T - 1.824 P), stresses in all members are then tabulated as functions of P, T, and loads in the redundant members and reaction H. Coefficient constants for the seven necessary equations are summarized as follows:

Panel	$\frac{u^2L}{EA}$	$\frac{HuL}{EA}$	$\frac{S'uL}{EA}$	$\frac{H^2L}{EA}$	$\frac{S'HL}{EA}$
1	167.358	-17.084	-37.796	total	total
2	189.720	-19.293	-42.709		
3	182.374	-18.561	-41.084		
4	182.374	-18.561	-39.054		
5	189.720	-19.293	146.371		
6	167.358	-16.799	111.368	60.478	(1.399T-67.732P)

Also for any two consecutive panels, summation u_1u_2L/EA etc., is equal to 1.141. From these constant coefficients, the equations are set up:

S_1:

$$167.358S_1 + 1.141S_2 - 17.084H - 37.796 P = 0 \qquad (1)$$

S_2:

$$1.141S_1 + 189.720S_2 + 1.141S_3 - 19.293H - 42.709 P = 0 \qquad (2)$$

S_3:

$$1.141S_2 + 182.374S_3 + 1.141S_4 - 18.561H - 41.084\ P = 0 \quad (3)$$

S_4:

$$1.141S_3 + 182.374S_4 + 1.141S_5 - 18.561H - 39.054\ P = 0 \quad (4)$$

S_5:

$$1.141S_4 + 189.720S_5 + 1.141S_6 - 19.293H + 146.371\ P = 0 \quad (5)$$

S_6:

$$1.141S_5 + 167.358S_6 - 16.799\ H + 111.388\ P = 0 \quad (6)$$

H:

$$-17.084S_1 - 19.293\ (S_2+S_5) - 18.561\ (S_3+S_4) - 16.799H$$
$$+ 60.478\ H - 67.732\ P + 1.399\ T = 0 \quad (7)$$

Solving, the following values are obtained:

$S_1 = (0.3432\ P - 0.002868\ T)$

$S_2 = (0.3403\ P - 0.002842\ T)$

$S_3 = (0.3403\ P - 0.002843\ T)$

$S_4 = (0.3354\ P - 0.002843\ T)$

$S_5 = (-0.6550\ P - 0.002842\ T)$

$S_6 = (-0.5434\ P - 0.002820\ T)$

$H = (1.1726\ P - 0.02828\ T)$

Final stresses for different members in the entire bottom lateral truss are obtained by substituting the above redundant stresses, and

TABLE V. STRESSES AND ENERGY COEFFICIENTS IN BOTTOM LATERAL SYSTEM DUE TO T

(1)	(2)	(3)				(4)		(5)	
		$S = AT + BP + CH + uS_{1,2,\ldots}$				S		$\frac{SL}{EA}\frac{\partial S}{\partial P}$	
Member	$\frac{L}{A}$	T	P	H	u	T	P	T	P
$L_{00}-L_{11}$	11.33	-0.0158	0.490	-0.833	$-0.827S_1$	0.01013	-0.771	-0.08849	6.73
$L_{11}-L_{22}$	11.33	-0.0158	0.857	-0.667	$-0.827S_2$	0.0054	-0.206	-0.01260	0.48
$L_{22}-L_{33}$	5.96	-0.0474	2.205	-0.500	$-0.827S_3$	-0.0309	+1.337	-0.24623	10.65
$L_{33}-L_{44}$	5.96	-0.0474	2.573	-0.333	$-0.827S_4$	-0.0356	1.905	-0.40419	21.60
$L_{44}-L_{55}$	11.33	-0.0316	0.980	-0.167	$-0.827S_5$	-0.0246	1.326	-0.36958	19.89
$L_{55}-L_{66}$	11.33	-0.0316	-0.123	----	$-0.827S_6$	-0.0293	0.326	-0.10822	1.20
L_0-L_1	11.33	+0.0158	-0.123	1.0	$-0.827S_1$	-0.01013	0.766	-0.08792	6.65
L_1-L_2	11.33	0.0158	-0.490	0.833	$-0.827S_2$	-0.0054	0.205	-0.01254	0.48
L_2-L_3	5.96	0.0474	-1.837	0.667	$-0.827S_3$	0.0309	-1.336	-0.24604	10.64
L_3-L_4	5.96	0.0474	-2.205	0.500	$-0.827S_4$	0.0356	-1.896	-0.40228	21.40
L_4-L_5	11.33	0.0316	-2.085	0.333	$-0.827S_5$	0.0246	-1.152	-0.32108	15.02
L_5-L_6	11.33	0.0316	-0.980	0.167	$-0.827S_6$	0.0293	-0.335	-0.11121	1.27
$L_{00}-L_0$	4.49	----	0.250	0.113	$-0.562S_1$	-0.0016	0.1897	-0.00136	0.16
$L_{11}-L_1$	3.61	----	0.250	0.113	$-0.562(S_1+S_2)$	----	-0.0016	----	----
$L_{22}-L_2$	3.61	----	0.250	0.113	$-0.562(S_2+S_3)$	----	----	----	----
$L_{33}-L_3$	3.61	----	0.250	0.113	$-0.562(S_3+S_4)$	----	0.0028	----	----
$L_{44}-L_4$	3.61	----	-0.750	0.113	$-0.562(S_4+S_5)$	----	-0.4379	----	0.69
$L_{55}-L_5$	3.61	----	-0.750	0.113	$-0.562(S_5+S_6)$	----	0.0560	----	----
$L_{66}-L_6$	4.49	----	----	----	$-0.562S_6$	0.0016	0.3050	0.00219	0.42

TABLE V. (CONT.) STRESSES AND ENERGY COEFFICIENTS IN BOTTOM LATERAL SYSTEM DUE TO T

(1)	(2)	(3)				(4)		(5)	
Member	$\frac{L}{A}$	$S = AT + BP + CH + uS_{1,2,\cdots}$				S		$\frac{SL}{EA}\frac{\partial S}{\partial P}$	
		T	P	H	u	T	P	T	P
$L_{00}-L_1$	74.65	----	-0.445	-0.201	S_1	0.00281	-0.337	-0.07069	8.48
L_0-L_{11}	74.65	----	----	----	S_1	-0.00287	0.343	-0.07348	8.78
$L_{11}-L_2$	85.97	----	-0.445	-0.201	S_2	0.00284	-0.340	-0.08301	9.94
L_1-L_{22}	85.97	----	----	----	S_2	-0.00284	0.340	-0.08301	9.94
$L_{22}-L_3$	85.97	----	-0.445	-0.201	S_3	0.00284	-0.340	-0.08301	9.94
L_2-L_{33}	85.97	----	----	----	S_3	-0.00284	0.340	-0.08301	9.94
$L_{33}-L_4$	85.97	----	-0.445	-0.201	S_4	0.00284	-0.345	-0.08423	10.23
L_3-L_{44}	85.97	----	----	----	S_4	-0.00284	0.335	-0.08179	9.65
$L_{44}-L_5$	85.97	----	1.333	-0.201	S_5	0.00284	0.442	0.10791	16.79
L_4-L_{55}	85.97	----	----	----	S_5	-0.00284	-0.655	0.15992	36.88
$L_{55}-L_6$	74.65	----	1.333	-0.201	S_6	0.00284	0.554	0.11745	22.91
L_5-L_{66}	74.65	----	----	----	S_6	-0.00284	-0.543	0.11512	22.01
								$\sum = -2.5514$	292.77

they are tabulated in column (4) of Table V. Again, differentiate the total strain energy in the bottom lateral truss caused by these final stresses with respect to P, and tabulate them in column (5) of Table V.

(d) Torque Distribution among the Trusses, Values of P and Q.--By summing up all the differential coefficients of the strain-energy in the entire bridge under the torque T with respect to P, the value of P is determined in terms of T:

Bridge elements	$\frac{\partial W}{\partial P}$
1. Top lateral with chord members:	$\frac{1}{E}(-5.898T + 363.166\,P)$
2. Portal bending:	$\frac{1}{E}(\text{-------} + 985.625\,P)$
3. Bottom lateral truss with chord members:	$\frac{1}{E}(-2.551\,T + 292.770\,P)$
4. 2 Vertical trusses with end posts, but not chord members:	$\frac{1}{E}(-5.238\,T + 148.598\,P)$
Total:	$\frac{1}{E}(-13.687\,T + 1790.159\,P)$

With the total differential coefficient of the strain-energy in the bridge with respect to P thus found, the value of P must be such that the total strain-energy stored in the bridge due to T is a minimum. That is,

$$\frac{\partial W}{\partial P} = 0$$

or

$$\frac{1}{E}(-13.687\,T + 1790.159\,P) = 0$$

and

$$P = 0.007645\ T$$

$$Q = 0.0588\ T - 1.824\ P$$

$$= 0.044882\ T$$

5. Stresses Caused by Load W at L_4. By combining the effects of w_1 and T, stresses for different members in the entire bridge can be found for any eccentric load W placed anywhere along the floor beam $L_{44} - L_4$. Thus assume W equal to 40000 lb. to be placed vertically at joint L_4,

$$w_1 = 20000 \text{ lbs. at } L_{44} \text{ and } L_4$$

$$T = 40000 \times 8.5 \text{ or } 340000 \text{ ft. lbs.}$$

$$P = 0.007645T \text{ or } 2600 \text{ lbs.}$$

$$Q = 0.044882T \text{ or } 15260 \text{ lbs.}$$

stresses for the whole bridge are tabulated in Table

Reactions:

$$H = (1.1726\ P - 0.028284\ T) = -6560 \text{ lbs.}$$

$$N_6 = H\,\frac{204}{1800} = 743 \text{ lbs.}$$

R_0, vertical reaction at L_0

$$= \left(\frac{1}{3}w_1 + \frac{1}{3}Q + \frac{1.824P}{4}\right) = 12{,}940 \text{ lbs.}$$

$$R_6 = \left(\frac{2}{3}w_1 + \frac{2}{3}Q + \frac{3}{4} \times 1.824\ P\right) = 27{,}060 \text{ lbs.}$$

$$R_{66} = -R_{00} = -\left(27{,}060 - \frac{2}{3}W\right) = -400 \text{ lbs.}$$

TABLE VI. STRESSES IN BRIDGE DUE TO W AT L_4

	Member	L, in.	A, sq.in.	$\frac{L}{A}$	Sw_1 due to w_1	S_t due to T	S (Sw_1+S_t)	$\frac{SL}{EA}$ Kips
Top Chords	$U_{11}-U_{22}$	300	43.33	6.92	-10260	+ 7718	- 2542	- 17.59
	$U_{22}-U_{33}$	300	43.33	6.92	-10320	+ 6764	- 3556	- 24.60
	$U_{33}-U_{44}$	300	43.33	6.92	-20550	+13917	- 6633	- 45.90
	$U_{44}-U_{55}$	300	43.33	6.92	-20496	+14871	- 5625	- 38.92
	U_1-U_2	300	43.33	6.92	-10260	- 7718	-17978	-124.41
	U_2-U_3	300	43.33	6.92	-10320	- 6759	-17079	-118.18
	U_3-U_4	300	43.33	6.92	-20550	-14083	-34633	-239.66
	U_4-U_5	300	43.33	6.92	-20496	-15040	-35536	-245.91
Struts	$U_{11}-U_1$	204	28.50	7.16	+ 340	----	+ 340	+ 2.43
	$U_{22}-U_2$	204	7.12	28.65	+ 640	+ 3	+ 643	+ 18.42
	$U_{33}-U_3$	204	7.12	28.65	+ 940	- 54	+ 886	+ 25.38
	$U_{44}-U_4$	204	7.12	28.65	+ 1320	+ 1185	+ 2505	+ 71.77
	$U_{55}-U_5$	204	28.50	7.16	+ 680	- 57	+ 623	+ 4.46
Top Diagonals	$U_{11}-U_2$	362.8	4.18	86.79	- 600	+ 577	- 23	- 2.00
	U_1-U_{22}	362.8	4.18	86.79	- 600	- 577	- 1177	-102.15
	$U_{22}-U_3$	362.8	4.18	86.79	- 540	+ 572	+ 32	+ 2.77
	U_2-U_{33}	362.8	4.18	86.79	- 540	- 582	- 1122	- 97.38
	$U_{33}-U_4$	362.8	4.18	86.79	- 1140	+ 678	- 462	- 40.09
	U_3-U_{44}	362.8	4.18	86.79	- 1140	- 478	- 1618	-140.42
	$U_{44}-U_5$	362.8	4.18	86.79	- 1220	- 1630	- 2850	-247.35
	U_4-U_{55}	362.8	4.18	86.79	- 1220	+ 1838	+ 618	+ 53.63

E = 1

TABLE VI. (CONT.) STRESSES IN BRIDGE DUE TO W AT L_4

Member		L, in.	A, sq.in.	$\frac{L}{A}$	S_{w_1} due to w_1	S_t due to T	S $(S_{w_1}+S_t)$	$\frac{SL}{EA}$ Kips
Verticals & Diagonals TT	$U_{11}-L_{00}$	477.9	49.45	9.66	- 8560	+ 7293	- 1267	- 12.24
	$U_{11}-L_{22}$	477.9	29.42	16.24	+ 8560	- 6531	+ 2029	+ 32.95
	$U_{33}-L_{22}$	477.9	32.36	14.77	- 8560	+ 6531	- 2029	- 29.97
	$U_{33}-L_{44}$	477.9	32.36	14.77	+ 8560	- 6531	+ 2029	+ 29.97
	$U_{55}-L_{44}$	477.9	29.42	16.24	+17120	-13062	+ 4058	+ 65.90
	$U_{55}-L_{66}$	477.9	49.45	9.66	-17120	+15347	- 1773	- 17.13
	$U_{11}-L_{11}$	372	24.35	15.28	----	----	----	----
	$U_{22}-L_{22}$	372	12.20	30.49	----	----	----	----
	$U_{33}-L_{33}$	372	24.35	15.28	----	----	----	----
	$U_{44}-L_{44}$	372	12.20	30.49	----	----	----	----
	$U_{55}-L_{55}$	372	24.35	15.28	----	----	----	----
Diagonals T	U_1-L_0	477.9	49.45	9.66	- 8560	- 7293	-15853	-153.14
	U_1-L_2	477.9	29.42	16.24	+ 8560	+ 6531	+15091	+245.08
	U_3-L_2	477.9	32.36	14.77	- 8560	- 6531	-15091	-222.89
	U_3-L_4	477.9	32.36	14.77	+ 8560	+ 6531	+15091	+222.89
	U_5-L_4	477.9	29.42	16.24	+17120	+13062	+30182	+490.15
	U_5-L_6	477.9	49.45	9.66	-17120	-15347	-32467	-313.63

E = 1

TABLE VI. (CONT.) STRESSES IN BRIDGE DUE TO W AT L_4

Member		L, in.	A, sq.in.	$\frac{L}{A}$	Sw_1 due to w_1	S_t due to T	S (Sw_1+S_t)	$\frac{SL}{EA}$ Kips
Verticals	U_1-L_1	372	24.35	15.28	----	----	----	----
	U_2-L_2	372	12.20	30.49	----	----	----	----
	U_3-L_3	372	24.35	15.28	----	----	----	----
	U_4-L_4	372	12.20	30.49	----	----	----	----
	U_5-L_5	372	24.35	15.28	----	----	----	----
Bottom Chords	$L_{00}-L_{11}$	300	26.48	11.33	+ 4880	+ 1439	+ 6319	+ 71.59
	$L_{11}-L_{22}$	300	26.48	11.33	+ 4940	+ 1300	+ 6240	+ 70.70
	$L_{22}-L_{33}$	300	50.40	5.96	+15420	- 7030	+ 8390	+ 50.00
	$L_{33}-L_{44}$	300	50.40	5.96	+15420	- 7151	+ 8269	+ 49.28
	$L_{44}-L_{55}$	300	26.48	11.33	+ 9900	- 4917	+ 4983	+ 56.45
	$L_{55}-L_{66}$	300	26.48	11.33	+ 9780	- 9115	+ 665	+ 7.53
	L_0-L_1	300	26.48	11.33	+ 4880	- 1453	+ 3427	+ 38.83
	L_1-L_2	300	26.48	11.33	+ 4940	- 1302	+ 3638	+ 41.22
	L_2-L_3	300	50.40	5.96	+15420	+ 7032	+22450	+133.80
	L_3-L_4	300	50.40	5.96	+15420	+ 7174	+22594	+134.66
	L_4-L_5	300	26.48	11.33	+ 9900	+ 5367	+15267	+172.97
	L_5-L_6	300	26.48	11.33	+ 9780	+ 9092	+18872	+213.82

E = 1

TABLE VI. (CONT.) STRESSES IN BRIDGE DUE TO W AT L_4

Member		L, in.	A, sq.in.	$\frac{L}{A}$	S_{w_1} due to w_1	S_t due to T	S $(S_{w_1}+S_t)$	$\frac{SL}{EA}$ Kips
Floor Beams	$L_{00}-L_0$	204	45.40	4.49	- 334	- 376	- 710	- 3.18
	$L_{11}-L_1$	204	56.47	3.61	- 628	- 4	- 632	- 2.28
	$L_{22}-L_2$	204	56.47	3.61	- 778	----	- 778	- 2.81
	$L_{33}-L_3$	204	56.47	3.61	- 968	+ 73	- 895	- 3.23
	$L_{44}-L_4$	204	56.47	3.61	-1074	-1138	-2212	- 7.98
	$L_{55}-L_5$	204	56.47	3.61	-1262	+ 145	-1117	- 4.03
	$L_{66}-L_6$	204	45.40	4.49	- 672	+ 362	- 310	- 1.39
Bottom Diagonals	$L_{00}-L_1$	362.8	4.86	74.65	+ 600	+ 78	+ 678	+ 50.61
	L_0-L_{11}	362.8	4.86	74.65	+ 600	- 83	+ 517	+ 38.59
	$L_{11}-L_2$	362.8	4.22	85.97	+ 520	+ 80	+ 600	+ 51.58
	L_1-L_{22}	362.8	4.22	85.97	+ 520	- 80	+ 440	+ 37.82
	$L_{22}-L_3$	362.8	4.22	85.97	+ 860	+ 80	+ 940	+ 80.81
	L_2-L_{33}	362.8	4.22	85.97	+ 860	- 80	+ 780	+ 67.05
	$L_{33}-L_4$	362.8	4.22	85.97	+ 860	+ 68	+ 928	+ 79.78
	L_3-L_{44}	362.8	4.22	85.97	+ 860	- 93	+ 767	+ 65.94
	$L_{44}-L_5$	362.8	4.22	85.97	+1040	+2115	+3155	+271.23
	L_4-L_{55}	362.8	4.22	85.97	+1040	-2668	-1628	-139.96
	$L_{55}-L_6$	362.8	4.86	74.65	+1200	+2405	+3605	+269.11
	L_5-L_{66}	362.8	4.86	74.65	+1200	-2378	-1178	- 87.94

E = 1

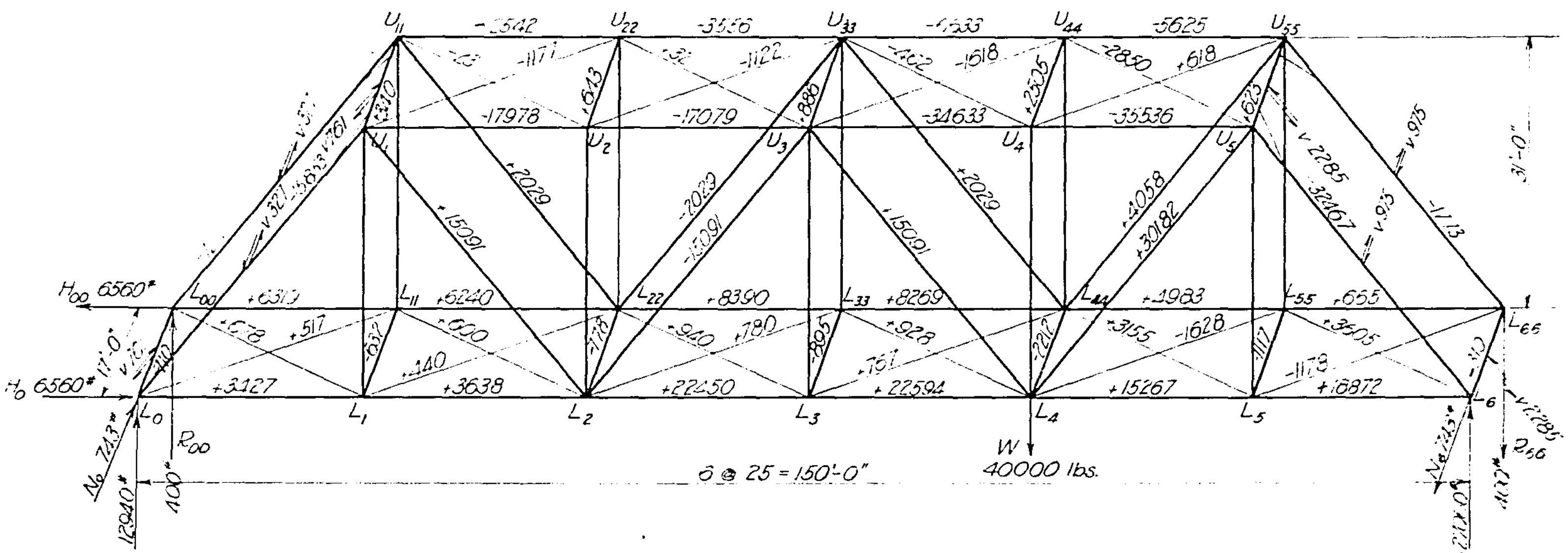


Fig. 27. Stresses, Shears & Reactions.

Shears in planes of portals with point of inflection at mid-section of end posts:

Left Portal, $U_{11}-L_0$

In end post $= \frac{1}{8}P = 325$ lbs.

In struts and floor beam,

$$\tfrac{1}{2}\left(\frac{P}{4} \times \frac{477.9}{204}\right) = 761 \text{ lbs.}$$

Right Portal, $U_{55}-L_6$

In end post, $\frac{3}{8}P = 975$ lbs.

In strut and floor beam,

$$761 \times 3 = 2283 \text{ lbs.}$$

Axial stresses in portal struts, end posts, and end floor beams include those due to portal action.

All stresses are shown in Fig. (27).

6. Displacements of the Bridge. From stresses obtained in the solution of article 4 (d), displacements of the whole structure can be either computed or found graphically. Since the general behavior of the entire structure is of more interest for a model test, deflections at all joints of the structure in space have been found by drawing Williot diagrams for all trusses. Deformations for all members are tabulated in the last column of Table VI.

Diagrams for trusses with redundant members are made by removing all the redundant members and reactions, and redundant members not used in the Williot diagrams are shown in dotted lines.

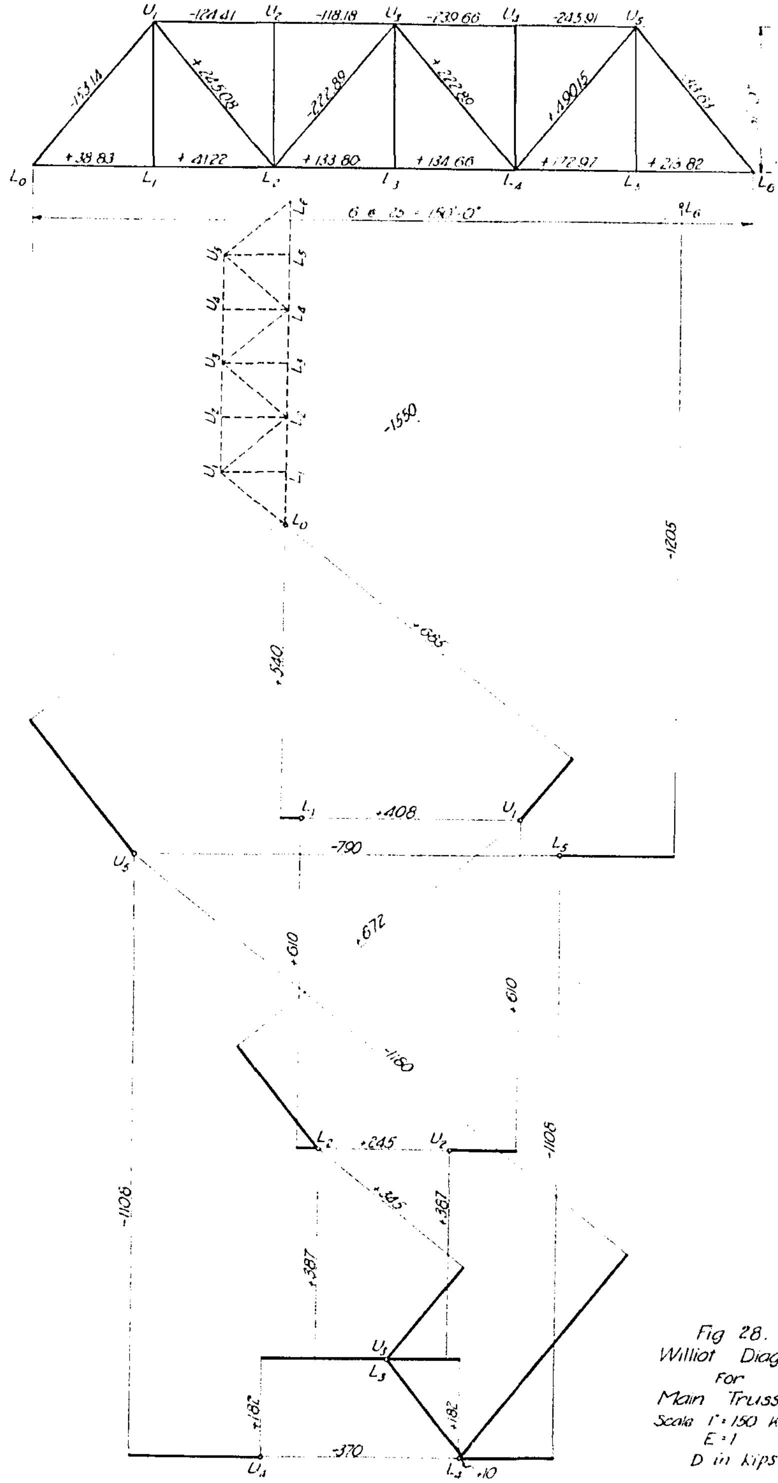
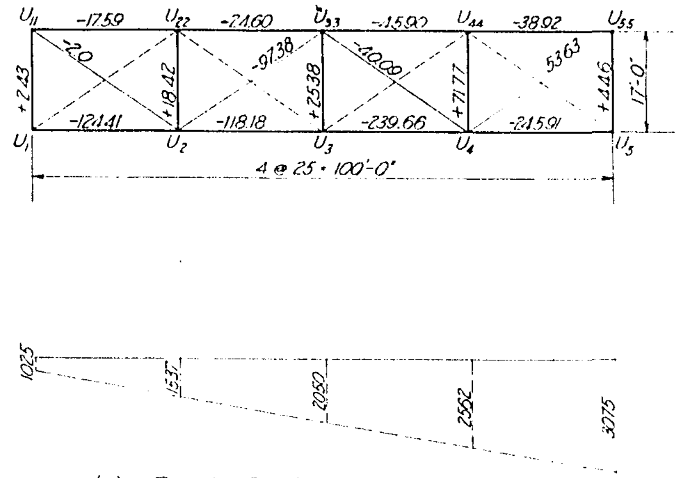
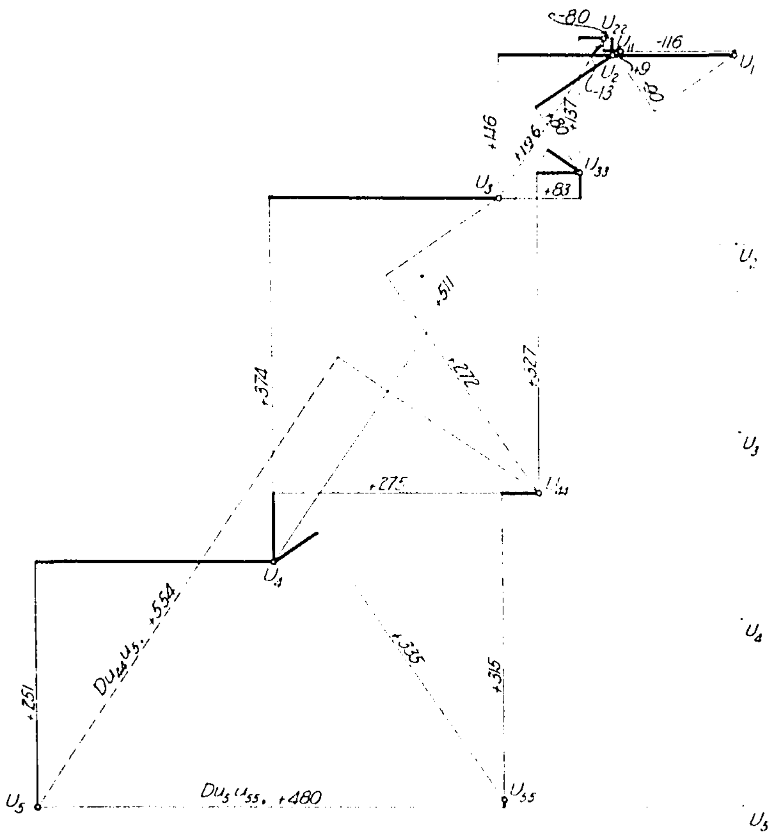


Fig 28.
Williot Diagram
For
Main Truss T.
Scale 1" = 150 Kips.
E = 1
D in Kips

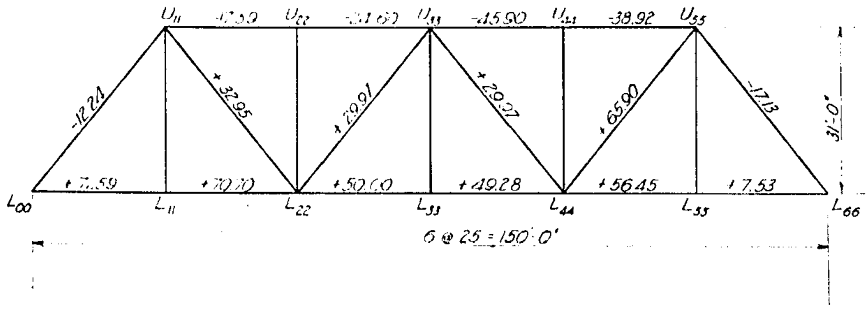


(a) Portal Displacements



(b) Truss Displacements

Fig 29.
Williot Diagram
for
Top Laterals
Scale 1" = 100 Kips
E = 1
D in Kips



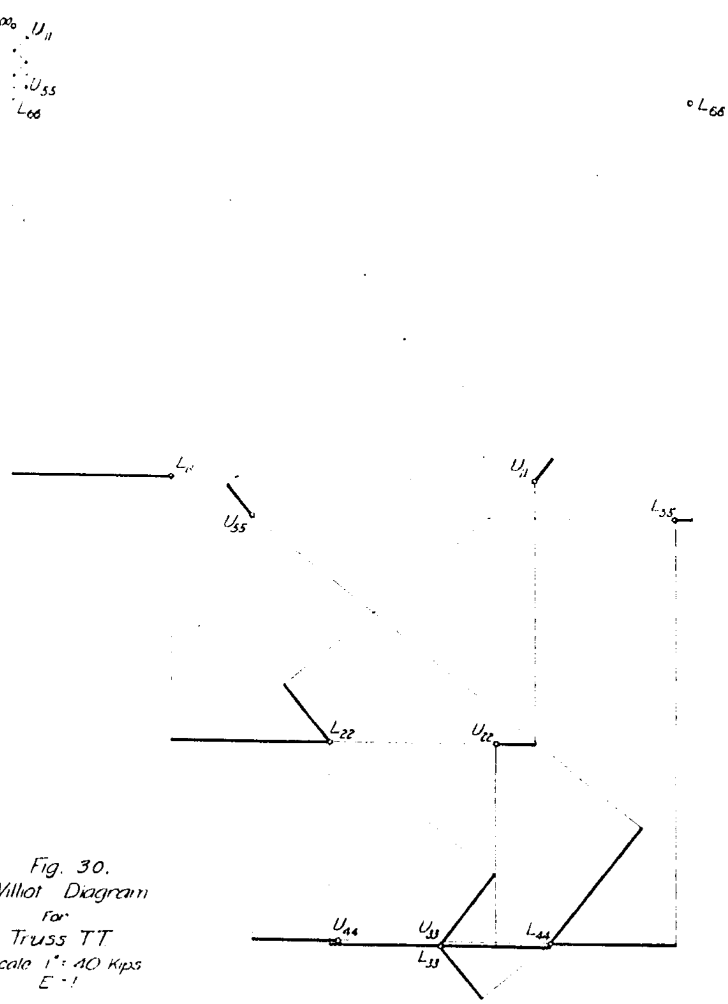
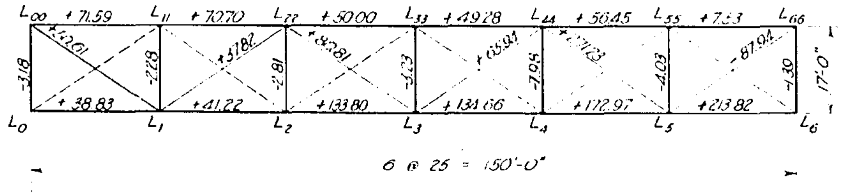


Fig. 30.
Williot Diagram
For
Truss TT
Scale 1" = 40 Kips
E = 1



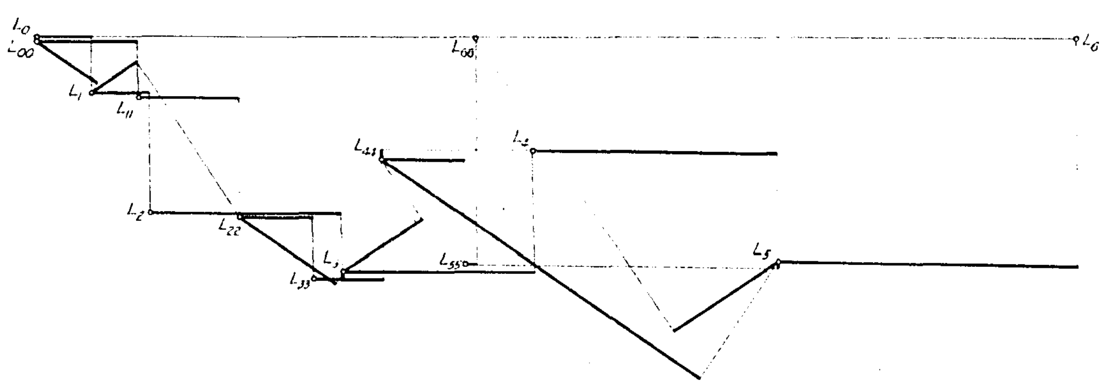


Fig. 31.
Williot Diagram
For
Bottom Laterals
Ref line L_{00} - L_0
Ref pt L_6
Scale 1" = 60 kips

Deflections for the top lateral must include those of the portals, which will deflect 1577/E inches when a unit load is applied at the corner of the top. The correction diagram for the top lateral truss can be made either by rotating the truss about U_1 so that U_5 will move horizontally and adding portal deflections, or to correct directly by rotating the truss about U_1 so as to make U_5 coincide with the displacement resulting from portal deflection. Portal deflections, however, are too large relative to member deformations for the scale of the diagram, so that the top lateral in this case is drawn as simply supported truss with portal deflections added later. Deflections in all directions are shown in Fig. (29). Also, an exaggerated deflection diagram is shown in Fig. (35).

Portal deflection:

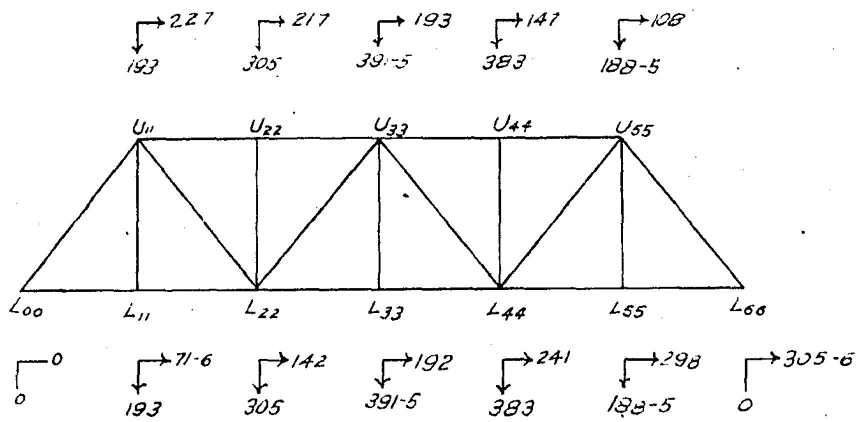
at left portal

$$d_l = \frac{P}{4} \times \frac{1577}{E} = \frac{1025}{E}$$

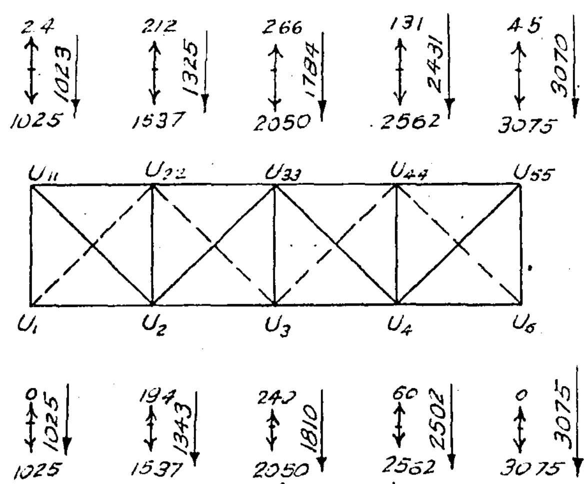
at right portal

$$d_r = \frac{3}{4} \times \frac{1577}{E} = \frac{3075}{E}$$

While it is true that in the plane of bottom laterals all joints are displaced toward the side of $L_0 - L_6$, joints in the plane of top laterals are moved far more in the same direction, and consequently, all transverse frames are twisted in the direction of the load W.



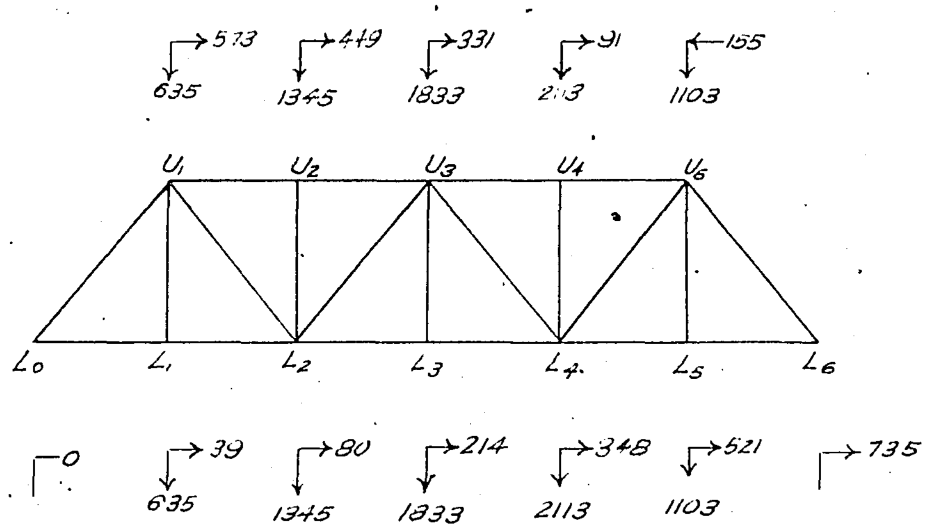
(a) Truss TT.



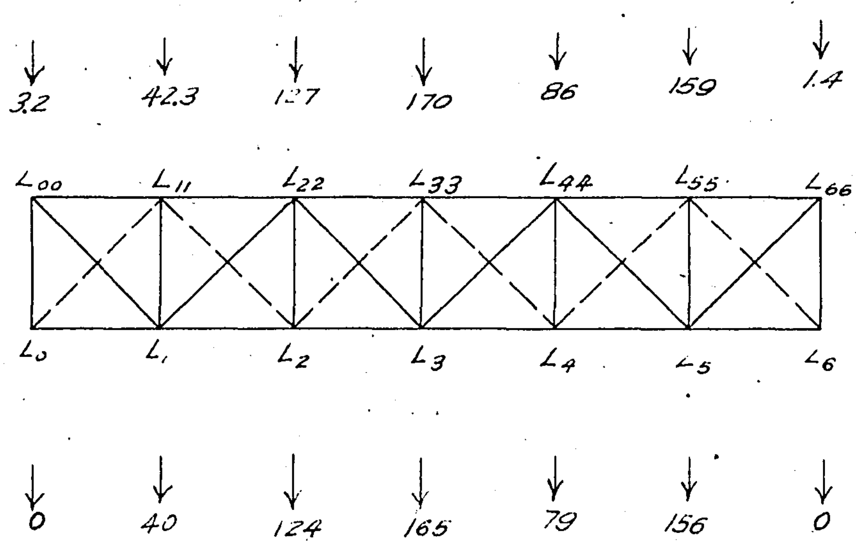
(b) Top Laterals.

Upward arrows for truss action,
downward ones for portals.
Net displacements shown on side.

Fig. 32.
Displacements of Truss TT
and Top Laterals.



(a) Truss T.



(b) Bottom Lateral System.
Fig. 33.
Displacements of Truss T
and Bottom Laterals.

Value in kips, E=1.

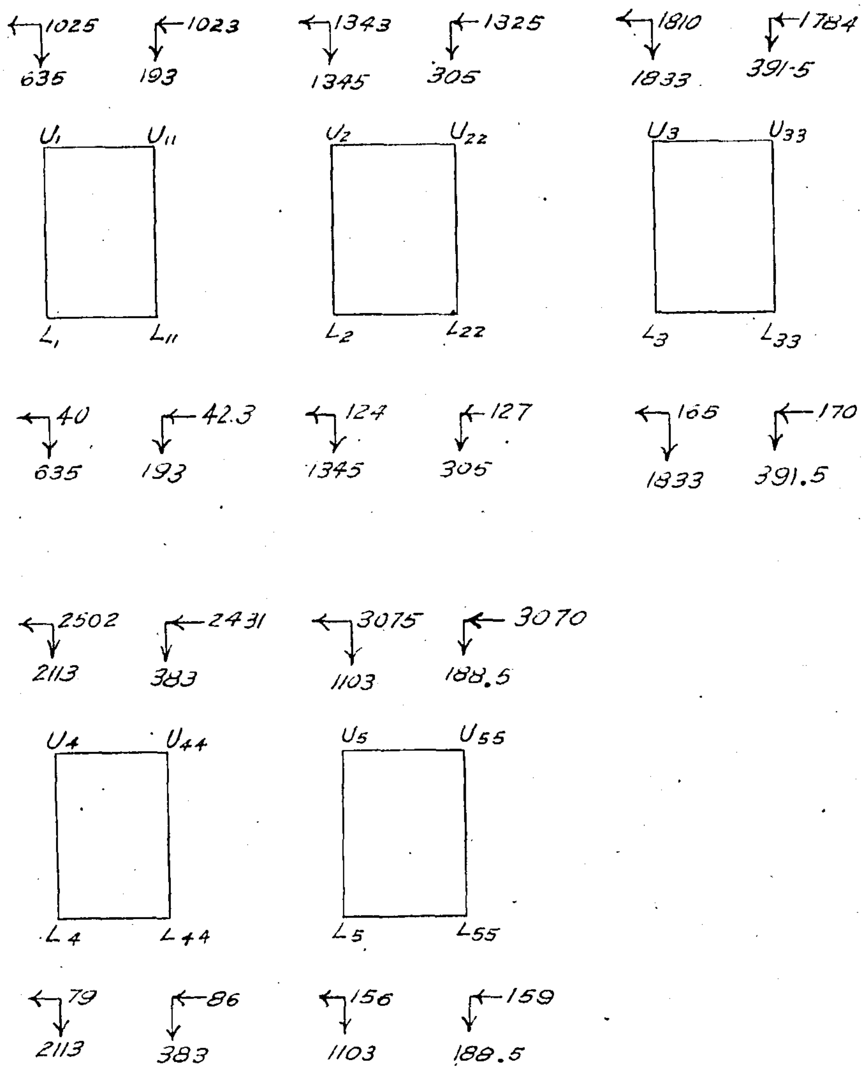


Fig. 34.
Displacements of
Transverse Frames.

Value in kips,
E=1

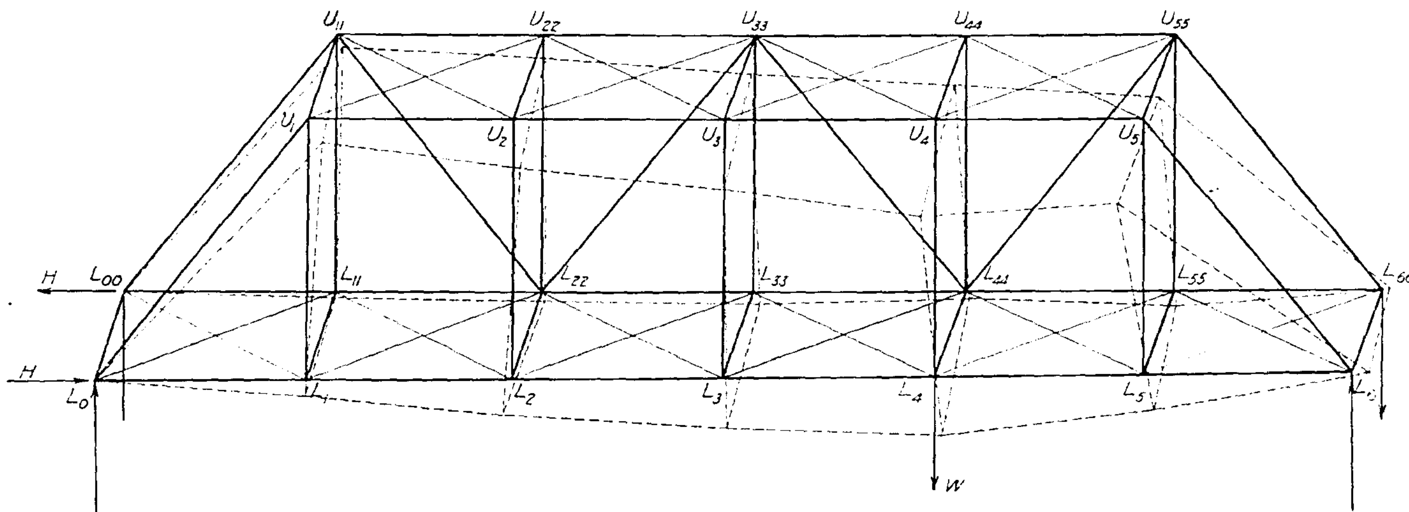


Fig. 35. Exaggerated Deflections.

7. <u>Secondary Stresses.</u> Secondary stresses can be computed for the space structure, after all the primary stresses have been found, on the same assumption as for planar trusses, that is, axial stresses are practically independent of the bendings of different members. Each member in space will bend in two directions about its principle axes in addition to the torsion it receives. Polar moment of inertia of structural shapes composed of narrow rectangles is equal to 1/3 of the summation of ld^3, in which l is the longer and d the shorter dimension respectively of each individual rectangle. Torsion stiffness is relatively negligible, and the amount of torsion each member would resist can be disregarded.

Of all the available methods for secondary stresses, Mohr's Semi-graphical method in this case is by far the simplest one, since the Williot diagrams necessary to set up all equations have already been made to find deflections.

(a) Loaded Vertical Truss.--Assuming all clockwise rotations as positive, all the values of D are scaled from Fig. ( ), measured in kips with E equal to unity. The moment at the end of any member is given by the equation,

$$M_{mn} = -2E\left(\frac{I_{mn}}{L_{mn}}\right)\left(2\theta_m - \theta_n - \frac{3D_{mn}}{L_{mn}}\right)$$

and the summation of all the moments about any joint m equal to zero gives the general equation,

TABLE VII. DATA FOR SECONDARY STRESS COMPUTATIONS IN LOADED TRUSS

	Member	A Sq.in.	L In.	I In.^{4}	c In.	c In.	$K = \frac{I}{L}$	$SR = \frac{3D}{L}$	3KR
Top Chords	U_1-U_2	43.33	300	2385.4	9.33*	9.54+	7.95	+ 6.10	+48.495
	U_2-U_3	43.33	300	2385.4	9.33*	9.54+	7.95	+ 3.87	+30.607
	U_3-U_4	43.33	300	2385.4	9.33*	9.54+	7.95	+ 1.82	+14.469
	U_4-U_5	43.33	300	2385.4	9.33*	9.54+	7.95	-11.08	-88.086
Bottom Chords	U_0-L_1	26.48	300	750.2	7.5	7.5	2.50	+ 5.40	+13.500
	L_1-L_2	26.48	300	750.2	7.5	7.5	2.50	+ 6.10	+15.250
	L_2-L_3	50.36	300	1076.4	7.5	7.5	3.59	+ 3.87	+13.893
	L_3-L_4	50.36	300	1076.4	7.5	7.5	3.59	+ 1.82	+ 6.534
	L_4-L_5	26.48	300	750.2	7.5	7.5	2.50	-11.08	-27.700
	L_5-L_6	26.48	300	750.2	7.5	7.5	2.50	-12.05	-30.125
Diagonals	U_1-L_0	49.45	477.9	2611.5	9.38*	9.49+	5.46	+ 4.30	+23.476
	U_1-L_2	29.42	477.9	805.4	7.5	7.5	1.68	+ 4.22	+ 7.089
	U_3-L_2	32.36	477.9	860.4	7.5	7.5	1.80	+ 2.16	+ 3.888
	U_3-L_4	32.36	477.9	860.4	7.5	7.5	1.80	+ 0.06	+ 0.108
	U_5-L_4	29.42	477.9	805.4	7.5	7.5	1.68	- 7.41	-12.449
	U_5-L_6	49.45	477.9	2611.5	9.38*	9.49+	5.46	- 9.78	-53.126
Verticals	U_1-L_1	24.35	372	112.86	5.28	5.28	0.30	+ 3.29	+ 0.987
	U_2-L_2	12.20	372	70.62	5.19	5.19	0.19	+ 1.98	+ 0.376
	U_3-L_3	24.35	372	112.86	5.28	5.28	0.30	———	———
	U_4-L_4	12.20	372	70.62	5.19	5.19	0.19	- 2.98	- 0.566
	U_5-L_5	24.35	372	112.86	5.28	5.28	0.30	- 6.37	- 1.911

*Top fibre +Bottom fibre D in Kips, E = 1

$$2\,\Sigma\,K\theta_m + K_{mn}\theta_n + K_{mo}\theta_o + K_{mp}\theta_p + \ldots. - 3\,\Sigma\,KR = 0$$

where, K is the I/L and R is the D/L value of any member; and mn, mO, etc. are members meeting at the joint m.

All the necessary data for setting up equations are tabulated in Table VII.

Joint L_0:

$$15.93\theta l_0 + 5.46\theta u_1 + 2.50\theta l_1 - 36.99 = 0 \qquad (1)$$

Joint L_0:

$$2.50\theta l_0 + 0.30\theta u_1 + 10.61\theta l_1 + 2.50\theta l_2 - 29.73 = 0 \qquad (2)$$

Joint U_1:

$$5.46\theta l_0 + 30.81\theta u_1 + 0.30\theta l_1 + 7.95\theta u_2 + 1.68\theta l_2 - 80.01 = 0 \qquad (3)$$

Joint U_2:

$$7.95\theta u_1 + 32.18\theta u_2 + 0.19\theta l_2 + 7.95\theta u_3 - 79.47 = 0 \qquad (4)$$

Joint L_2:

$$1.68\theta u_1 + 2.50\theta l_1 + 0.19\theta u_2 + 19.53\theta l_2 + 1.80\theta u_3 + 3.59\theta l_3$$
$$- 40.50 = 0 \qquad (5)$$

Joint U_3:

$$7.95\theta u_2 + 1.80\theta l_2 + 39.61\theta u_3 + 0.30\theta l_3 + 7.95\theta u_4 + 1.80\theta l_4$$
$$- 49.08 = 0 \qquad (6)$$

Joint L_3:

$$3.59\theta_{\ 2} + 0.30\theta u_3 + 14.96\theta l_3 + 3.59\theta l_4 - 20.43 = 0 \qquad (7)$$

Joint U_4:

$$7.95\theta u_3 + 32.18\theta u_4 + 0.19\theta l_4 + 7.95\theta u_5 + 74.19 = 0 \qquad (8)$$

Joint L_4:

$$1.30\theta u_3 + 3.59\theta l_3 + 0.19\theta u_4 + 19.53\theta l_4 + 1.68\theta u_5 + 2.50\theta l_5 + 34.05 = 0 \qquad (9)$$

Joint U_5:

$$7.95\theta u_4 + 1.68\theta l_4 + 30.81\theta u_5 + 0.30\theta l_5 + 5.46\theta l_6 + 155.58 = 0 \qquad (10)$$

Joint L_5:

$$2.50\theta l_4 + 0.30\theta u_5 + 10.61\theta l_5 + 2.50\theta l_6 + 59.73 = 0 \qquad (11)$$

Joint L_6:

$$5.46\theta u_5 + 2.50\theta l_5 + 15.93\theta l_6 + 83.25 = 0 \qquad (12)$$

Solution gives:

$\theta u_1 = +1.8229$	$\theta l_0 = +1.3623$
$\theta u_2 = +1.7129$	$\theta l_1 = +2.1343$
$\theta u_3 = +1.2088$	$\theta l_2 = +1.2588$
$\theta u_4 = -1.6178$	$\theta l_3 = +1.3163$
$\theta u_5 = -3.9642$	$\theta l_4 = -1.1551$
	$\theta l_5 = -4.4992$
	$\theta l_6 = -3.1598$

Fiber stresses for any member can be found from the relation,

$$f_s = \frac{M_{mn}C_{mn}}{I_{mn}} = -2\left(\frac{C}{L}\right)_{mn} (2\theta_m + \theta_n - 3R_{mn})$$

Heavy bending will occur in members about the loaded joint, namely:

U_3-U_4, U_4-U_5, U_4-L_4, U_5-L_5, and L_4-L_5, the three chord members and two verticals. Secondary stresses in these members are tabulated as follows:

TABLE VIII. SECONDARY STRESSES IN LOADED TRUSS

Location	$\frac{2c}{L}$	$(3R - 2\theta_m - \theta_n)$	$f_s = \frac{2c}{L}(3R-2\theta_m-\theta_n)$ x1000lbs/in.^{2}
U_3-U_4		+ 1.0202	+ 63.5 - 64.9
U_4-U_3	0.0622 top	+ 3.8468	-239.3 +244.6
U_4-U_5	0.0636 bottom	- 3.8802	-241.3 +246.8
U_5-U_4		- 1.5338	+ 95.4 - 97.5
U_4-L_4	0.0279	+ 1.4107	± 39.3
L_4-U_4		+ 0.9480	± 26.4
U_5-L_5	0.02839	+ 6.0576	±171.9
L_4-L_5	0.05	- 4.2706	±213.5
L_5-L_4	0.05	- 0.9265	± 46.3

(b) Top Lateral Truss.--Again using Mohr's Semi-graphical method for computing secondary stresses in the top laterals, all the D values are scaled from Fig. ( ). For redundant diagonals, the values of D are obtained by measuring the perpendicular distances to the direction of those redundant members, and these are shown in Fig. by dotted lines.

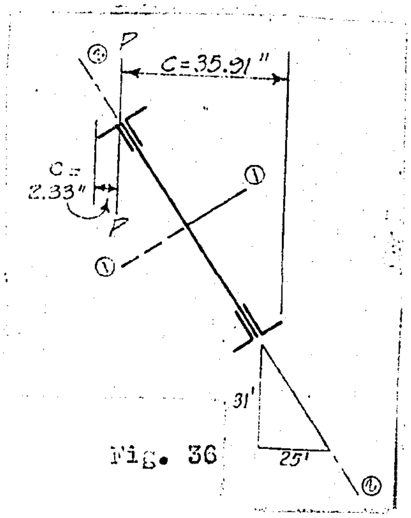


Fig. 36

Portal Strut

Web $53\frac{1}{8}" \times 3/8"$

4 Ls $3" \times 3" \times 3/8"$

$I_{11} = 10546.3$ in.^{4}

$I_{22} = 17.2$ in.^{4}

$I_{pp} = 12353.9$ in.^{4}

From $2\Sigma K\theta_m + K_{mn}\theta_n + K_{m0}\theta_0 + \ldots\ldots - 3\Sigma KR = 0$,

The following equations are set up:

Joint U_1:

$$137.324\theta u_1 + 60.559\theta u_{11} + 8.084\theta u_2 + 0.019\theta u_{22} + 103.388 = 0 \qquad (1)$$

Joint U_{11}:

$$60.559\theta u_1 + 137.324\theta u_{11} + 0.019\theta u_2 + 8.084\theta u_{22} + 104.424 = 0 \qquad (2)$$

Joint U_2:

$$8.084\theta u_1 + 0.019\theta u_{11} + 32.55\theta u_2 + 0.069\theta u_{22} + 8.084\theta u_3 + 0.019\theta u_{33} - 11.808 = 0 \qquad (3)$$

TABLE IX. DATA FOR SECONDARY STRESS COMPUTATIONS IN TOP LATERAL TRUSS

Member		A Sq.in.	L In.	I $In.^4$	$K = \frac{I}{L}$	$3R = \frac{3D}{L}$	3KR
Top Chords	$U_{11}-U_{22}$	43.33	300	2425.2	8.084	− 0.130	− 1.051
	$U_{22}-U_{33}$	43.33	300	2425.2	8.084	+ 1.370	+11.075
	$U_{33}-U_{44}$	43.33	300	2425.2	8.084	+ 3.270	+26.434
	$U_{44}-U_{55}$	43.33	300	2425.2	8.084	+ 3.150	+25.464
	U_1-U_2	43.33	300	2425.2	8.084	----	----
	U_2-U_3	43.33	300	2425.2	8.084	+ 1.460	+11.802
	U_3-U_4	43.33	300	2425.2	8.084	+ 3.740	+30.220
	U_4-U_5	43.33	300	2425.2	8.084	+ 2.510	+20.281
Struts	$U_{11}-U_1$	28.5	204	12353.9	60.559	− 1.707	−102.420
	$U_{22}-U_2$	7.12	204	14.0	0.069	− 0.117	− 0.008
	$U_{33}-U_3$	7.12	204	14.0	0.069	+ 1.220	+ 0.084
	$U_{44}-U_4$	7.12	204	14.0	0.069	+ 4.044	+ 0.279
	$U_{55}-U_5$	28.5	204	12353.9	60.559	+ 7.059	+427.486
Top Lateral Diag.	$U_{11}-U_2$	4.18	362.8	6.8	0.019	+ 0.074	+ 0.001
	U_1-U_{22}	4.18	362.8	6.8	0.019	− 0.744	− 0.014
	$U_{22}-U_3$	4.18	362.8	6.8	0.019	+ 1.621	+ 0.031
	U_2-U_{33}	4.18	362.8	6.8	0.019	+ 0.661	+ 0.013
	$U_{33}-U_4$	4.18	362.8	6.8	0.019	+ 4.225	+ 0.080
	U_3-U_{44}	4.18	362.8	6.8	0.019	+ 2.249	+ 0.043
	$U_{44}-U_5$	4.18	362.8	6.8	0.019	+ 4.581	+ 0.087
	U_4-U_{55}	4.18	362.8	6.8	0.019	+ 2.770	+ 0.052

D in Kips

Joint U_{22}:

$$0.019\theta u_1 + 8.084\theta u_{11} + 0.069\theta u_2 + 32.550\theta u_{22} + 0.019\theta u_3 + 8.084\theta u_{33} - 10.033 = 0 \tag{4}$$

Joint U_3:

$$8.084\theta u_2 + 0.019\theta u_{22} + 32.550\theta u_{33} + 0.069\theta u_{33} + 8.084\theta u_4 + 0.019\theta u_{44} - 42.356 = 0 \tag{5}$$

Joint U_{33}:

$$0.019\theta u_2 + 8.084\theta u_{22} + 0.069\theta u_3 + 32.550\theta u_{33} + 0.019\theta u_4 + 8.084\theta u_{44} - 37.686 = 0 \tag{6}$$

Joint U_4:

$$8.084\theta u_3 + 0.019\theta u_{33} + 32.55\theta u_4 + 0.069\theta u_{44} + 8.084\theta u_5 + 0.019\theta u_{55} - 51.098 = 0 \tag{7}$$

Joint U_{44}:

$$0.019\theta u_3 + 8.084\theta u_{33} + 0.069\theta u_4 + 32.550\theta u_{44} + 0.019\theta u_5 + 8.084\theta u_{55} - 52.307 = 0 \tag{8}$$

Joint U_5:

$$8.084\theta u_4 + 0.019\theta u_{44} + 137.324\theta u_5 + 60.559\theta u_{55} - 447.864 = 0 \tag{9}$$

Joint U_{55}:

$$0.019\theta u_4 + 8.084\theta u_{44} + 60.559\theta u_5 + 137.324\theta u_{55} - 453.002 = 0 \tag{10}$$

Solving, the following values are obtained:

$\theta u_{11} = -0.5381$ $\qquad$ $\theta u_1 = 0.5304$

$\theta u_{22} = +0.2165$ $\qquad$ $\theta u_2 = +0.2345$

$\theta u_{33} = +0.8495$ $\quad$ $\theta u_3 = +1.0477$

$\theta u_{44} = +0.8040$ $\quad$ $\theta u_4 = +0.7569$

$\theta u_{55} = +2.2740$ $\quad$ $\theta u_5 = +2.2136$

Heavy bending occurs in portal struts and chord members at the ends of the truss.

TABLE X. SECONDARY STRESSES IN TOP LATERAL TRUSS

Location	$\frac{2c}{L}$	$(3R-2\theta_m - \theta_n)$	$f_s = \frac{2c}{L}(3R-2\theta_m-\theta_n)$ x 1000 lbs./in.^{2}
U_3-U_4	0.07, 0.08	+0.8877	± 62.1 ± 71.0
U_4-U_3	0.07, 0.08	+1.1785	± 82.5 ± 94.3
U_4-U_5	0.07, 0.08	-1.2174	∓ 85.2 ∓ 97.4
U_5-U_4	0.07, 0.08	-2.6817	∓187.7 ∓214.5
U_5-U_{55}	$\begin{cases}0.0114\\0.1759\end{cases}$	+0.2974	$\begin{cases}+3.4\\-52.3\end{cases}$

Upper sign shows fibers on the inside of the top lateral truss.

For top chords, c on top = 10.5", $\frac{2c}{L} = 0.07$
on lower side c = 12", $\frac{2c}{L} = 0.08$.

(c) Transverse Frame at L_4.--Displacement of the entire structure causes the transverse frames to distort so that they are no longer rectangular. That is, the distortion of members will deform the transverse frames and cause bending moments in the latter. Consider each rectangular frame as supported at the joints of the main trusses, moments are found from the slope deflection method with the D/L values obtained from rotations of the vertical members relatively to the strained positions of the floor beam. Take the transverse frame U_4-L_4-U_{44}-L_{44} where the load is applied:

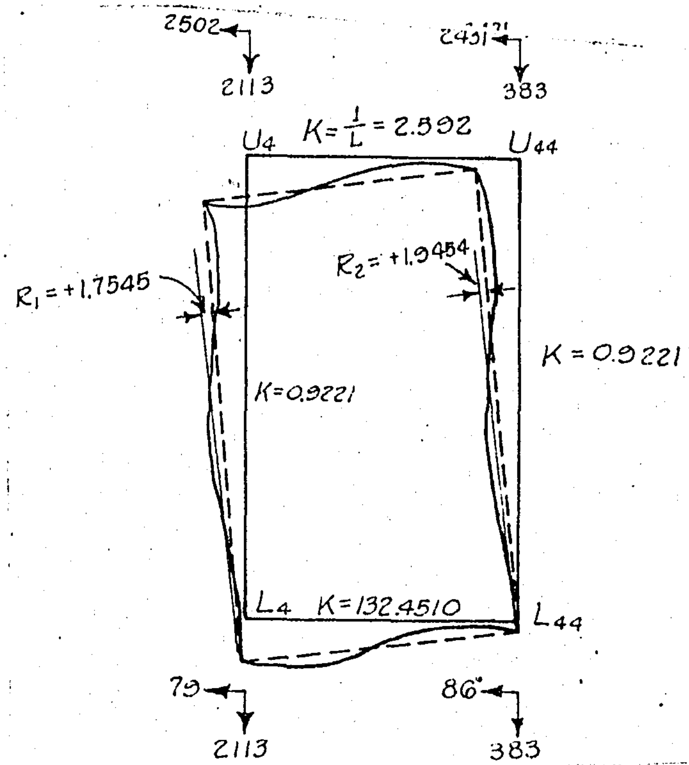


Fig. 37

Bending in Transverse Frame.

For detail of construction see prototype of model design Part III.

The vertical members are not stressed, and neglect the change in length of the floor beam L_4-L_{44}, the D/L values of the verticals are:

$$\text{Rotation of floor beam} = \frac{(2113-383)}{204} = 8.4804$$

$$R_1 = -\frac{(2502-79)}{372} + 8.4808 = +1.7545$$

$$R_2 = 8.4808 - \frac{(2431-86)}{372} = +1.9454$$

R for floor beam and strut = 0

Expressing moments in the general equation,

$M_{mn} = 2K_{mn}(-2\theta_m - \theta_n + 3R_{mn})$, and summation moments about each joint equal to zero, the four equations are:

Joint U_4:

$$-7.0284\theta u_4 - 0.9221\theta l_4 - 2.5921\theta u_{44} + 5.4413 = 0 \qquad (1)$$

Joint L_4:

$$-0.9221\theta u_4 - 266.7462\theta l_4 - 132.4510\theta l_{44} + 5.4413 = 0 \qquad (2)$$

Joint U_{44}:

$$-2.5921\theta u_4 - 7.0284\theta u_{44} - 0.9221\theta l_{44} + 6.0214 = 0 \qquad (3)$$

Joint L_{44}:

$$-132.4510\theta l_4 - 0.9221\theta u_{44} - 266.7462\theta l_{44} + 6.0214 = 0 \qquad (4)$$

Solving, $\theta u_4 = +0.5295 \qquad \theta u_{44} = +0.6595$

$\theta l_4 = +0.0112 \qquad \theta l_{44} = +0.0148$

and moments by substitution are:

$Mu_4 l_4 = -Mu_4 u_{44} = +8909$ in. lbs.

$Ml_4 - u_4 = -M l_4 - l_{44} = +9864$ in. lbs.

$Mu_{44} - U_{44} = -Mu_{44} - u_4 = +9583$ in. lbs.

$Ml_{44} - u_{44} = Ml_{44} - l_4 = +10772$ in. lbs.

Fiber stresses for vertical $U_4 - L_4$, $c = 6.5$ in.

$I = 343.04$ in.^{4}

$U_4 - L_4$. $\quad f_s = 8909 \times \frac{6.5}{343.04} = \pm 168$ lbs./in.^{2}

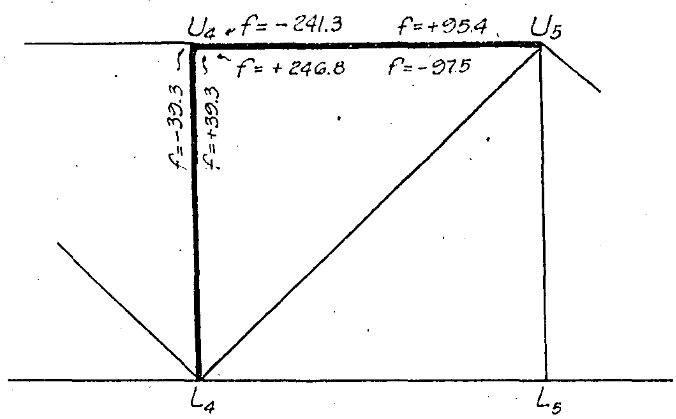
$L_4 - U_4$. $\quad f_s = 9864 \frac{c}{I} = \pm 187$ lbs./in.^{2}

(d) Combined Bending Stresses.--Combining the effects of all bending moments in three different planes, the resultant fiber stresses of any member can be found. Only one or two critical members will be shown here, others could be combined easily from the data supplied in foregoing stress computations. For bottom chords and floor beams, secondary stresses in the bottom lateral truss must be determined in addition to the computations already made, before combined fiber stresses can be obtained.

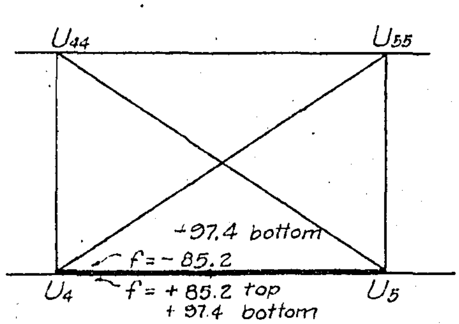
Thus, in Fig. (38) top chord U_4-U_5 will have fiber stresses of 246.8 lbs. per sq. in. tension at the lower fiber in the plane of the main truss, and 97.4 lbs. per sq. in. tension at the outer fiber in the plane of top lateral truss. The maximum bending stress is 344.4 lbs. per sq. in., ? which is relatively small in comparison with the primary stress - 800 lbs. per sq. in. in this member.

In the same way, maximum fiber stress in the vertical U_4-L_4 at the end U_4 develops at the fiber on the inner side of the transverse frame U_4-U_{44} and on the side of U_5 in the plane of the main truss. The total fiber stress, however, is 207.3 lbs. per sq. in. in tension.

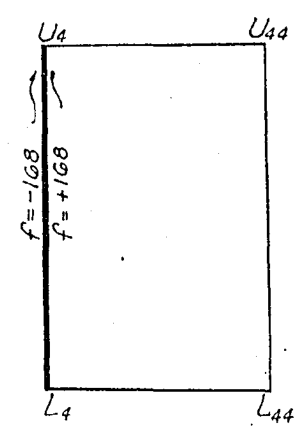
The same analysis applies to all members composing the entire bridge, and since stresses from bending is relatively of much smaller magnitude in comparison with axial stresses, no further details will be given.



(a) Secondary stresses in plane of loaded truss.



(b) Secondary stresses in plane of top lateral truss.



(c) In plane of transverse frame.

Fig. 38.
Combined secondary stresses in members around U_4, and at the end of U_4.
f, lbs./ sq. in.

IV. MODEL ANALYSIS

A. INTRODUCTION

The Principle of Similitude first appeared in Newton's Principia, Book II in 1687 in a brief and general form. But this principle has not been widely discussed in technical papers until within the last twenty years.

One of the earlier papers which combined this principle with engineering problems was E. Buckingham's "Model Experiments and the Form of Empirical Equations" in 1915 (56). In this paper, the author applied the Principle of Similitude to certain dynamic problems in engineering.

As to structural analysis with models designed by the theory of similitude, in 1931 three models have been designed and tested for the proposed San Francisco-Oakland Suspension Bridge under the direction of Professor G. E. Boggs and R. E. Davis and H. E. Davis (53). As far as bridge structures are concerned, this is the first model analysis of some particular prototype, although in 1930 A. Bull has published a paper which described the construction of a bridge truss with the stiffness of each member taken into consideration by a wire spring (55).

For space models, various models for aeroplane structures have been recently constructed and tested in different countries. Most of the results are either not published or inaccessible. In 1924 and 1925, Professor A. J. S. Pippard of Bristol, England, has made a series of

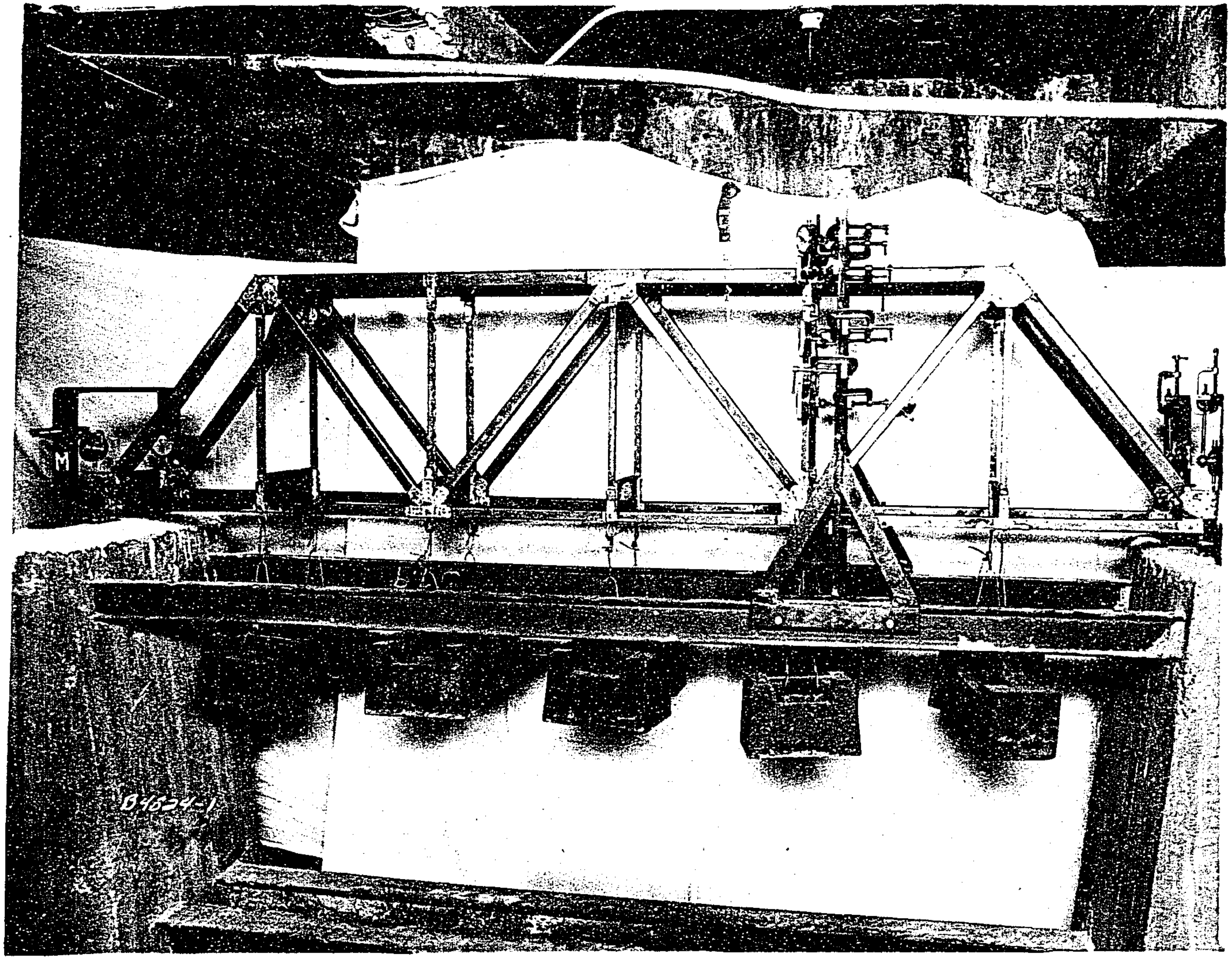


PLATE II.

BRIDGE MODEL AND LOADING FRAME

tests on an aeroplane structure model (29, 30, 31). In his tests, Professor Pippard has studied the stress distribution and the similarity between solids and a redundantly braced space structure.

A survey of the literature in the field of model analysis of structures, indicates that all the heretofore constructed models may be classified into three types:

1. Planar models that are reduced by the theory of similitude from certain prototypes, and measured results are then compared with the theoretical computations.

no prototypes

2. Space models that are designed independently and with a geometric figure approximating that of the actual structure; however, different dimensions may be used. Computations are made directly for the designed models and the results compared with tests on the model.

3. Space models that are in every way identical with the actual structure, that is, a small scale reproduction which is designed according to the principles of similitude.

Since the analysis of structures with models of three dimensions has had limited application as yet, the account of the model design and tests to be presented in this paper could, therefore, accomplish the following purposes:

1. The applicability of a space model which is reduced by the theory of similitude from a definite prototype for engineering research, and for various illustrative purposes.

2. The practicability of a space model design and construction,

which would cost only a small fraction of a precise miniature reproduction. The relatively easy construction and the low cost of a space model should undoubtedly extend its use.

3. The most direct value of this design, however, is to check up the author's computations which were presented in Chapter III for a prototype. On account of the limited literature concerning space structures and frames, the author hopes to make a definite contribution to the knowledge of this type of structural analysis.

B. THEORY OF SIMILITUDE FOR STRUCTURAL MODELS IN THREE DIMENSIONS

The present discussion of the Theory of Similitude will be limited to structures in general and to steel bridges in particular. For models in any other field of engineering technical literature could be consulted elsewhere.

The fundamental principles that govern the design of models are the geometric and mechanical similarity between the model and its prototype. The geometric similarity requires that the model should be similar to the prototype geometrically, that is, unit stresses will be the same when the model is made of the same material as the prototype, and unit strains and angular deformations are the same irrespective of materials. Linear dimensions should not affect the structural behavior.

The mechanical similarity requires that forces arising from different effects must bear a fixed ratio between the prototype and the model, that is, the force reduction factor shall not change.

The following symbols are employed for the prototype:

L = linear dimension

A = area of cross section

E = modulus of elasticity

I = moment of inertia

F = force or total stress

W = weight, or gravitational force

M = moment of force

R = radius of curvature

e = unit strain

s = unit stress

n = linear scale-reduction factor, i.e., $\frac{L}{L_1}$

w = density of material

Further let L_1, A_1, E_1, etc., the same symbols with subscript 1 represent corresponding terms for the model.

The weight of each member varies as the product of volume and density, and volume in turn varies as the cube of a linear dimension.
Thus weight ratio will be:

$$\frac{W}{W_1} = \frac{F}{F_1} = \frac{wL^3}{w_1 L_1^3} = \frac{w}{w_1}\, n^3$$

The total internal force or stress of an elastic member varies directly as area, unit deformation, and modulus of elasticity. The elastic force ratio will be:

$$\frac{F}{F_1} = \frac{AEe}{A_1 E_1 e_1} = \frac{L^2 E}{L_1^2 E_1} = n^2 \frac{E}{E_1}, \text{ since } e = e_1$$

Unit stresses between prototype and model:

$$\frac{S}{S_1} = \frac{F}{A} \Big/ \frac{F_1}{A_1} = \frac{F}{F_1} \times \frac{A_1}{A} = n^2 \frac{E}{E_1} \times \frac{L_1{}^2}{L^2} = \frac{E}{E_1}$$

Moment of a force varies as the product of force and distance, or

$$\frac{M}{M_1} = \frac{F \times L}{F_1 \times L_1} = \frac{L^3}{L_1{}^3} = n^3$$

Radius of curvature varies only as distance, and is therefore a linear factor,

$$\frac{R}{R_1} = \frac{L}{L_1} = n$$

The moment of inertia from beam deflections has the relation,

$$M = \frac{EI}{R}$$

or

$$\frac{M}{M_1} = \frac{EI}{R} \Big/ \frac{E_1 I_1}{R_1}$$

whence

$$\frac{I}{I_1} = \frac{MRE_1}{M_1 R_1 E} = n^4 \frac{E_1}{E}$$

A summary of the various scale-reduction factors expressed as ratios of prototype to model are as follows:

1.	Length	$L/L_1 = n$
2.	Area	$A/A_1 = n^2 E_1/E$
3.	Force or total stress	$F/F_1 = n^2$
4.	Moment of inertia	$I/I_1 = n^4 E_1/E$
5.	Moment of force	$M/M_1 = n^3$

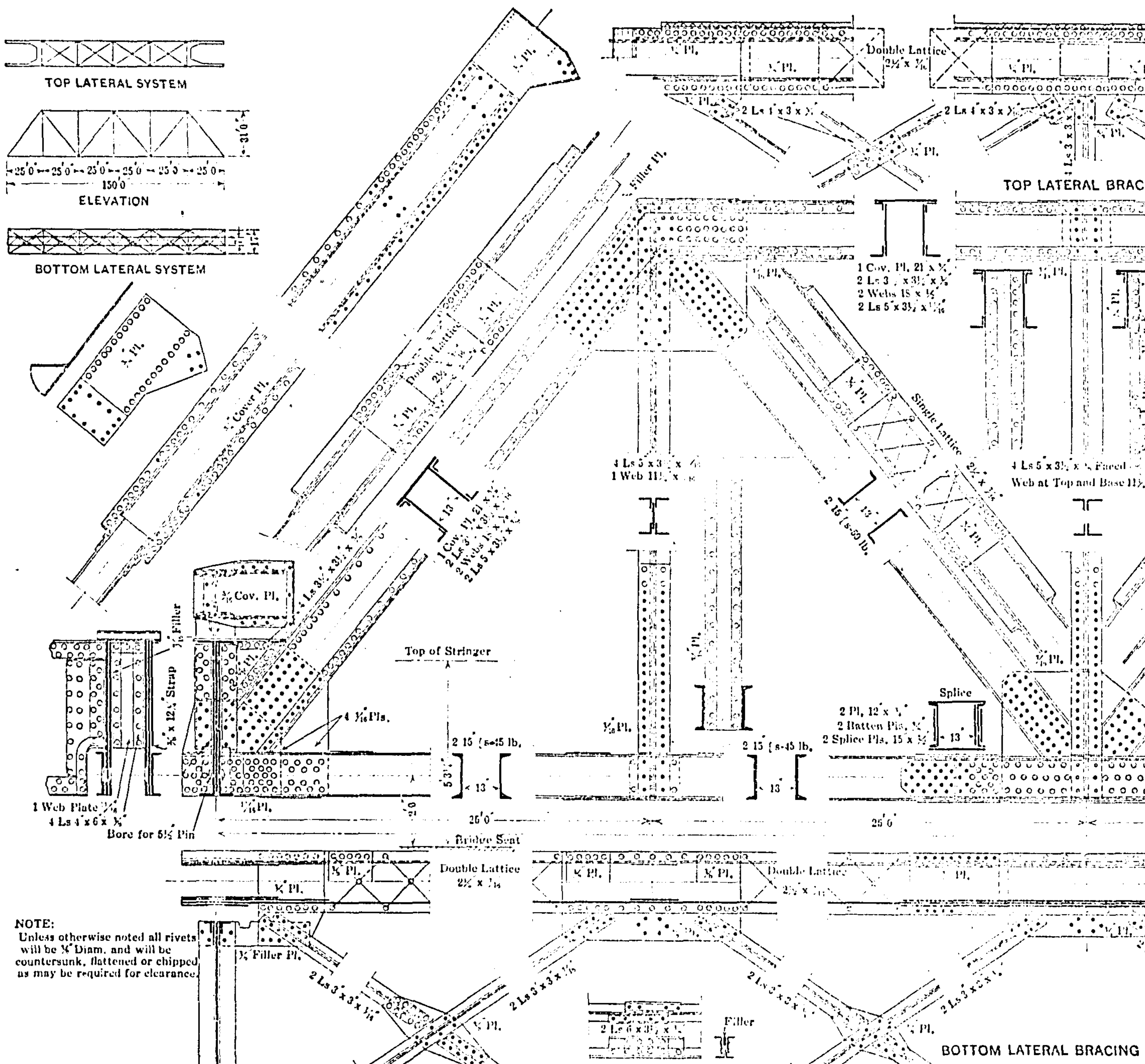


PLATE III.

MAIN TRUSS DETAILS OF PROTOTYPE

Plates III,IV and V from "Secondary Stresses in Bridges," Cecil Vivian
American Society of Civil Engineers, Vol.89,p.64,19

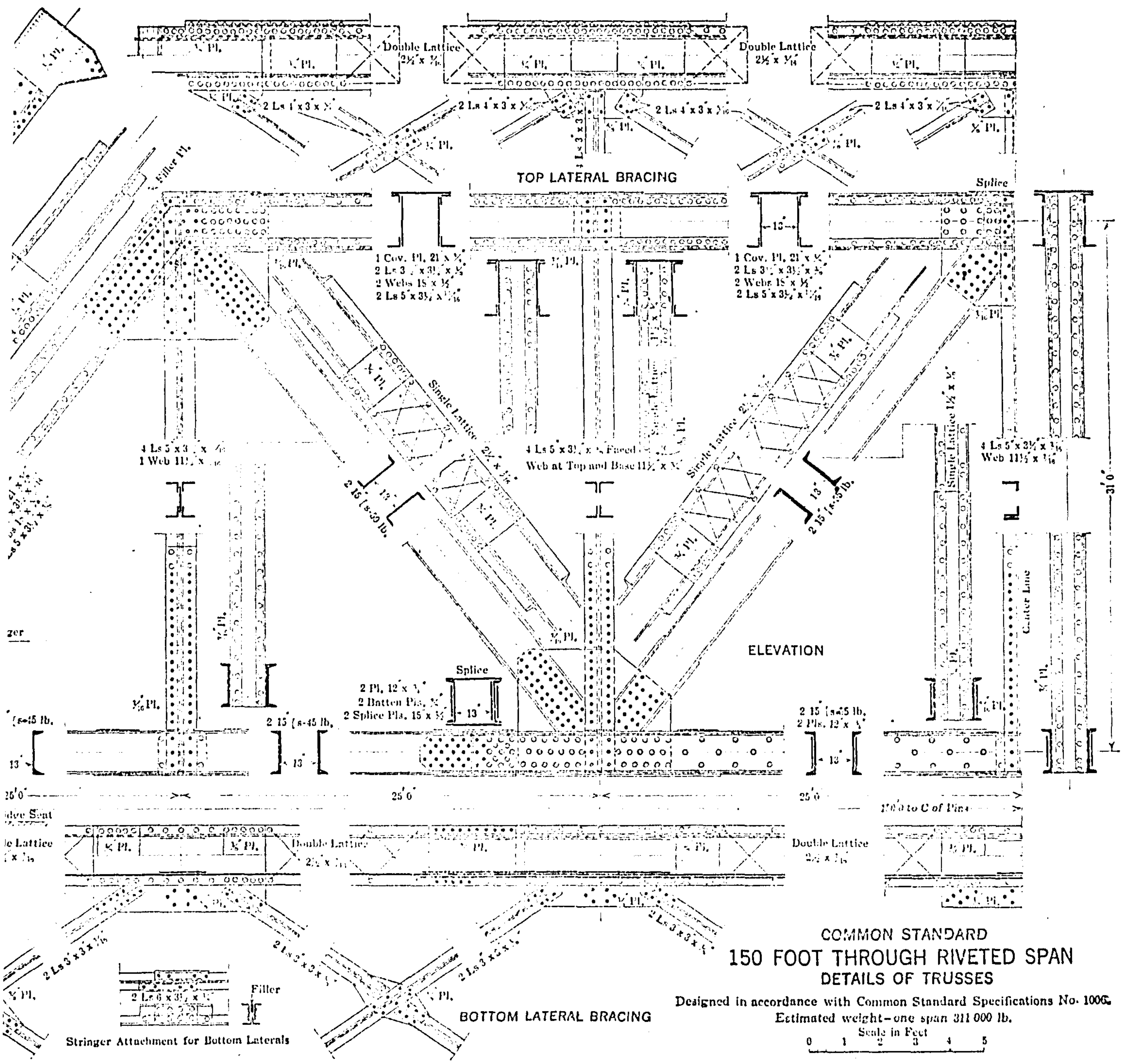


PLATE III.

MAIN TRUSS DETAILS OF PROTOTYPE

Secondary Stresses in Bridges," Cecil Vivian von Abo, Transactions -
an Society of Civil Engineers, Vol.89,p.64,1926

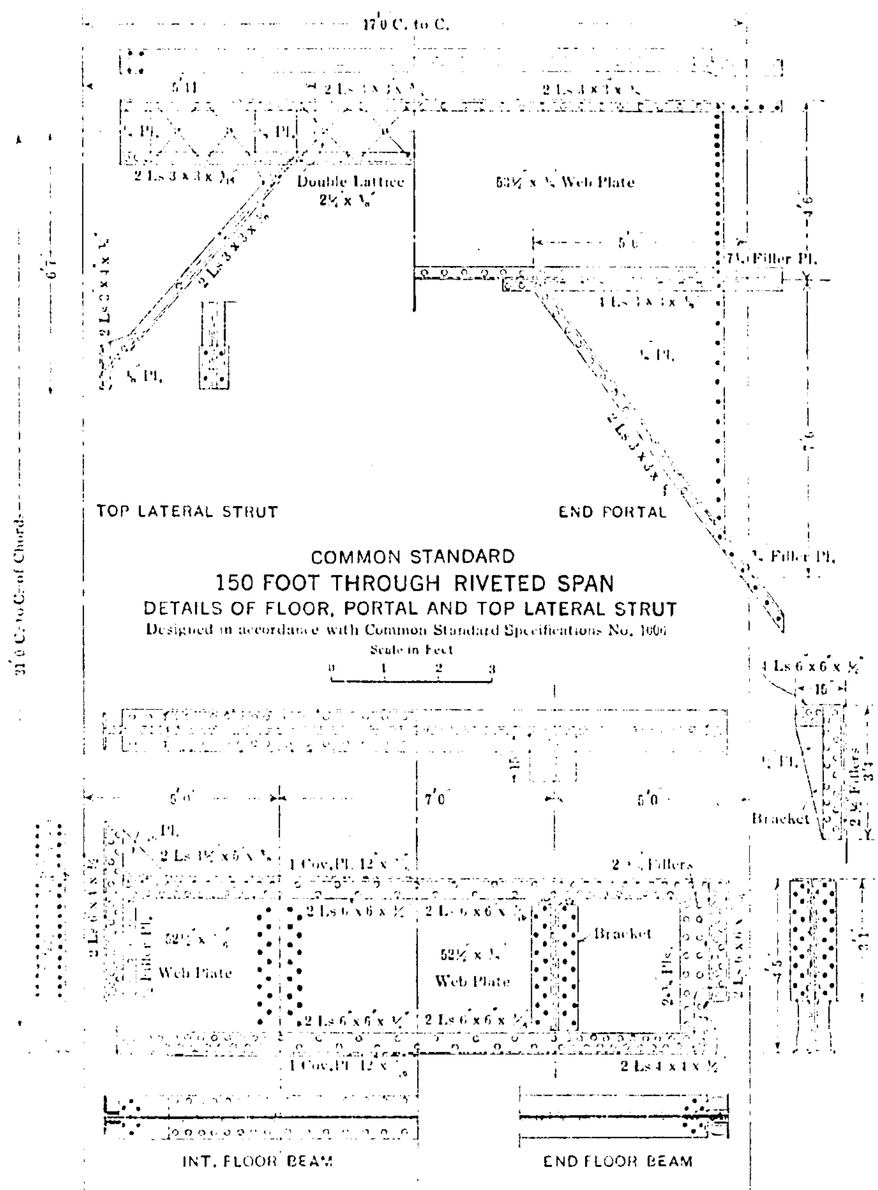


PLATE IV.

DETAILS OF FLOOR, PORTAL AND TOP LATERAL
STRUT OF PROTOTYPE

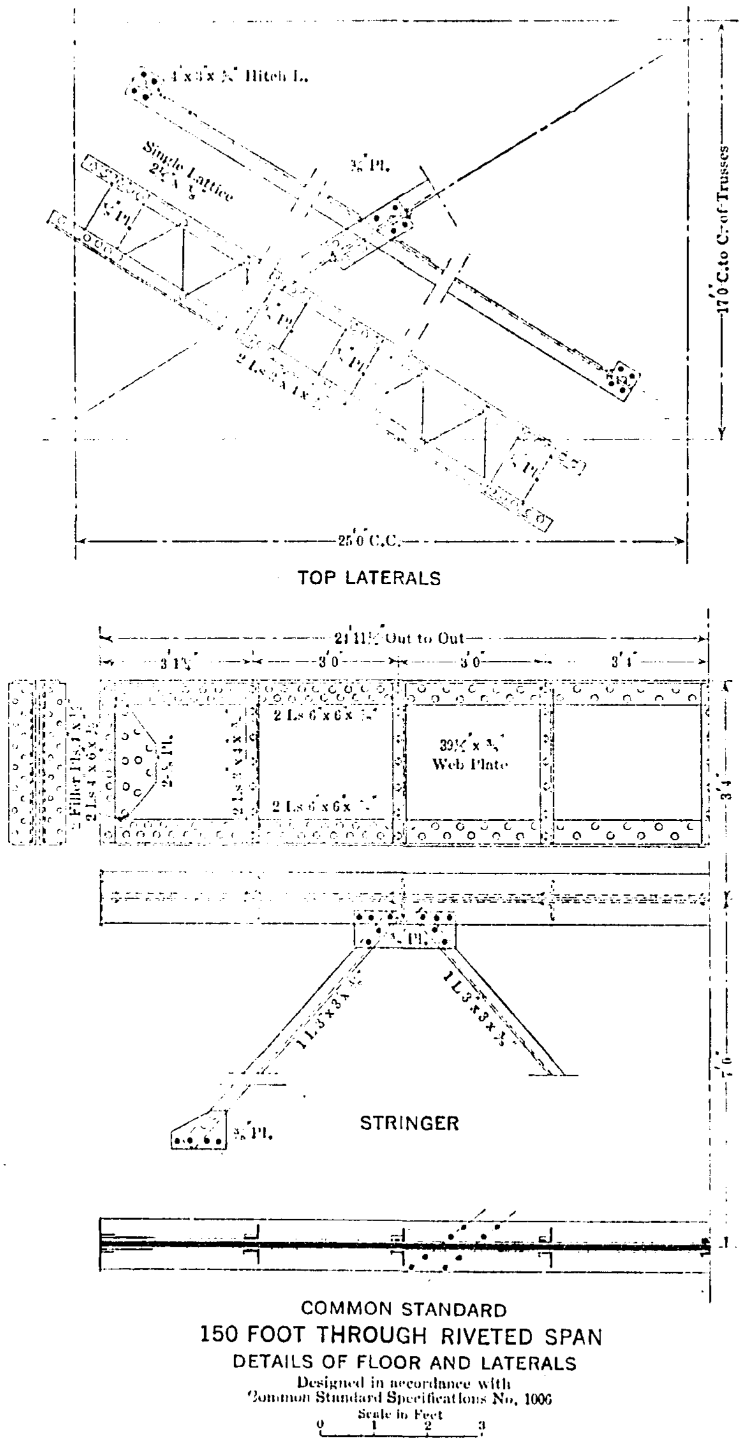


PLATE V.

DETAILS OF FLOOR AND LATERALS OF PROTOTYPE

6. Density $w/w_1 = 1/n \times E/E_1$

7. Unit strain $e/e_1 = 1$

8. Unit stress $s/s_1 = E/E_1$

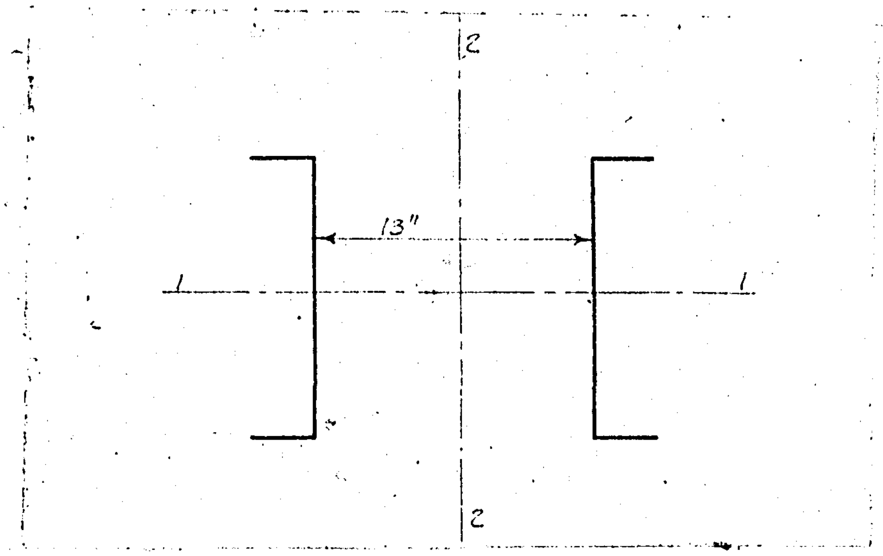
C. DESIGN OF A MODEL OF THREE DIMENSIONS FROM A 150-FOOT STANDARD BRIDGE

1. Materials used and Method of Design. The prototype of the model is a 150-foot, single track, through railway bridge. Considering all factors from a practical point of view, it was decided to use a linear reduction factor of $n = 20$, in order to make the model convenient for laboratory tests. Unquestionably, the section areas of the model members are to be similar, if not identical, to those of the prototype so as to render similar structural behavior. That is, some of the model members must have some hollow sections and must be braced together one way or another.

Materials for ordinary small scale models as celluloid or paper cannot be used for this model, because neither celluloid nor paper could be carved or braced so that separate sections can behave as one unit. With practical factors considered, materials to be used for this model are thin plate steel, or what is commercially known as "Black Sheet Iron."

When sheet steel can be used, properties of the material and other details of construction will be taken up later. The procedure of design is as follows:

Take the bottom chord as an example, the prototype section and approximate model section are shown in Fig. (39) (a) and (b).



(a) Bottom Chord in End Panels of Prototype

Section:

2 15" 45 lbs. Channels

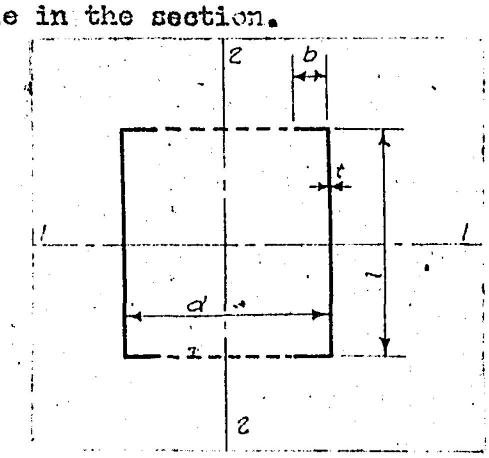
Area = 26.48 sq. in.

I_{11} = 750.2 in.^{4}

I_{22} = 1427.0 in.^{4}

$J = \Sigma \frac{1}{3} Ld^3$

Where d = thickness, l = longer dimension of any small rectangle in the section.



(b) Bottom Chord in End Panel of Model

Required properties:

$$\text{Area} = \frac{A}{n^2} = \frac{26.48}{(20)^2} = 0.0662 \text{ sq. in.}$$

$$I_{11} = \frac{750.2}{(20)^4} = 0.006688 \text{ in.}^4$$

$$I_{22} = \frac{1427.0}{(20)^4} = 0.008918 \text{ in.}^4$$

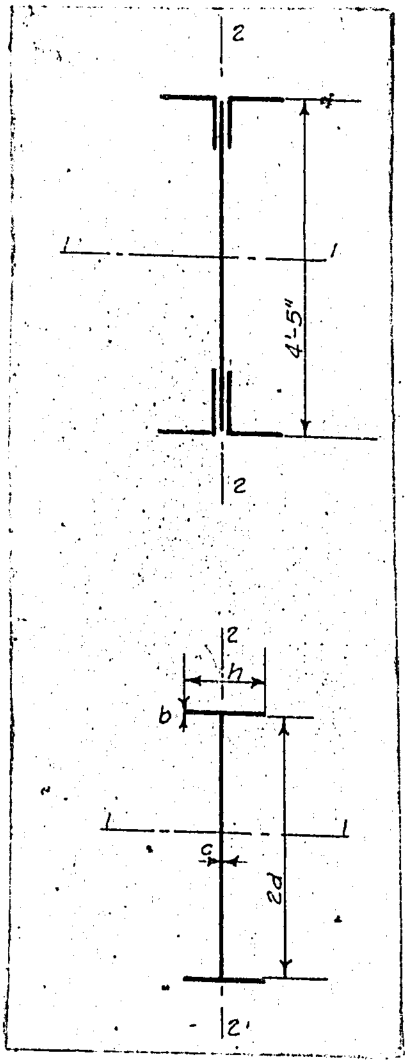
$$J_1 = \frac{J}{(20)^4}$$

Fig. 39. Bottom Chord Section

Thus there are four conditions to be fulfilled, the factor of E can be omitted since the material has been assumed to have the same modulus of elasticity as the prototype. Also the computations for the prototype can be made for any value of E, it is immaterial to consider the value of E in either case unless tests are to be made for the prototype.

The four equations for the four necessary conditions are to be found from the dimensions, l, t, d, and b, as shown in (b). Thus for each member, there are four equations to be solved. At first, Horner's Method of cubic equations was tried, but practical factors in the construction of the model make an exact solution quite unnecessary. In the first place, it is highly desirable to keep the back to back width of the model bottom chords uniform, and any exact solution to create a slight difference in width is unwarranted. In the second place, the torsion stiffness of any structural shape is negligibly small in comparison with its stiffness for bending in the planes of the trusses. Lastly, the exact solution often results in a thickness of the sheet steel not commercially used, for odd gages of this material are generally furnished only on special order.

All these considerations have convinced the author to adopt a cut and trial slide rule design, with torsion stiffness disregarded and a few percentages of error allowed in design. The design of the end floor beam will illustrate the procedure:



(a)

Prototype end floor beam:

Section: 4 Ls 6 x 6 x 9/16

1 web $52\frac{1}{2}$ x 3/8

Area, A = 45.40 sq. in.

I_{11} = 20416.47 in.^{4}

I_{22} = 181.47 in.^{4}

Model section for end floor beam:

Area required,

$$A = \frac{45.40}{(20)^2} = 0.113500 \text{ sq. in.}$$

$$I_{11} = \frac{20416.47}{(20)^4} = 0.127603 \text{ in.}^4$$

$$I_{22} = \frac{181.47}{(20)^4} = 0.00113406 \text{ in.}^4$$

Fig. 40. Design of End Floor Beam

b Assumed in in.	h $\left(\frac{6I_{22}}{b}\right)^{\frac{1}{3}}$	A_f 2bh	A $(A-A_f)$	c Assumed	d $\frac{Aw}{2c}$	I_{11} $A_f(d\ \frac{b}{2})^2$	$c\frac{(2d)^3}{12}$	Total
0.03125	0.602	0.0377	0.0758	0.03125	1.213	0.0570	0.0370	0.094
				0.0375	1.012	0.0399	0.0259	0.0658
				0.025	1.516	0.0883	0.0579	0.1462
0.025	0.651	0.032	0.0810	0.025	1.620	0.0848	0.0791	1.1639
				0.03125	1.295	0.0546	0.0675	0.1221

Final Section used, b = 0.025", h = 0.65", c = 0.03125", 2d = 2.80"

i.e. 2 flange plates of (0.65)(0.025)

1 web (2.80)(0.03125)

Area = (2)(0.65)(0.025) (2.80)(0.03125) = (0.0325 0.0875) = 0.1200 sq.in.

I_{22} = 0.001144 1 in^4

I_{11} = (0.0325)(1.4125)^2 (2.8)^3(0.03125) = (0.0650 0.0572) = 0.1222 in^4

The reason for assuming both b and c is to avoid computed thicknesses that differ from commercial sizes. Design by cut and trial, although fairly tedious, is by far the simplest method in comparison with the solution of a few simultaneous polynomials as ideal values require.

In the first few members, often a few trials are required, but after a little experience, a fairly approximate guess could be made from the prototype regarding the necessary thickness to be used. In the end floor beam design, the limitation of available materials has introduced errors of a small percentage. Thus in the section adopted, the area is increased by 5.7%; I_{11} decreased by 4.2%; and I_{22} increased by 0.9% from the theoretical values. This method of design has been used for all members of the entire bridge.

2. Design of Members. With the delivery of materials, a few members had to be redesigned to allow for the differences of catalog thicknesses and those checked by micrometer. Redesigned members are starred in the table for sections. Available materials used are listed:

Gage Number	Catalog thickness, in in.	Actual thickness, in in.
14	0.07815	0.0748
18	0.05	0.0487
20	0.0375	0.0375
22	0.03125	0.03125
24	0.0250	0.025
26	0.01875	0.01875

Sheet steel thinner than gage 26 would be much better for some lateral bracing members, but due to its impracticability to work with, other thicknesses have been used with allowance of some errors. In the following table, locations of members are referred to Fig. (41).

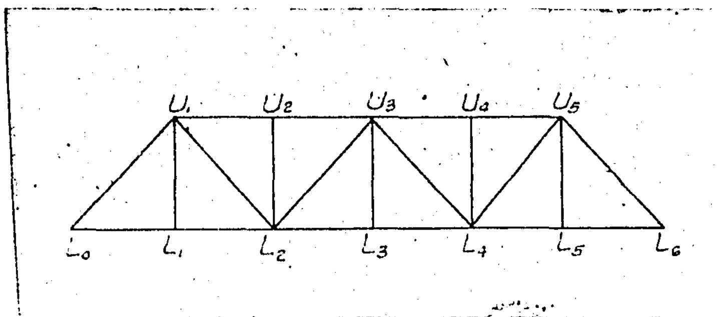
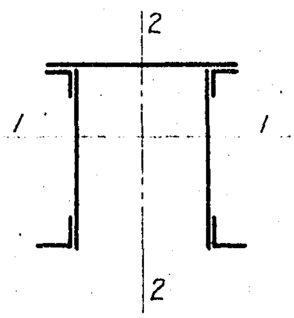
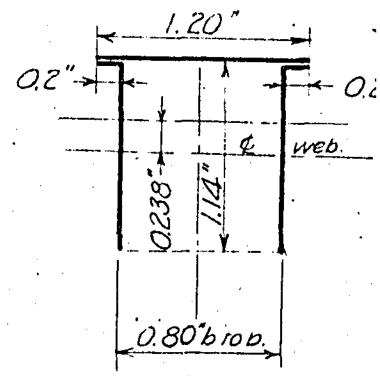
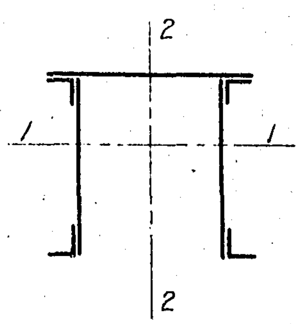
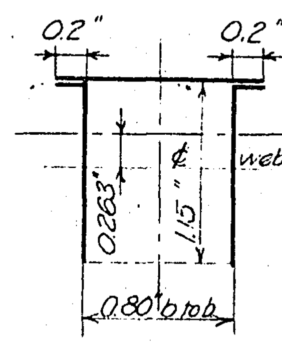


Fig. 41

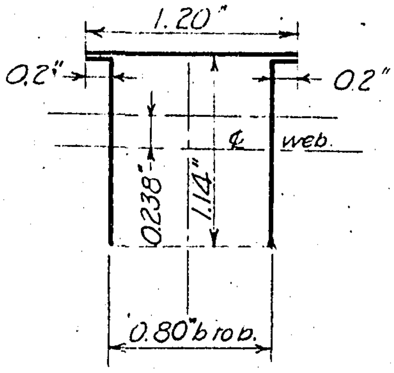
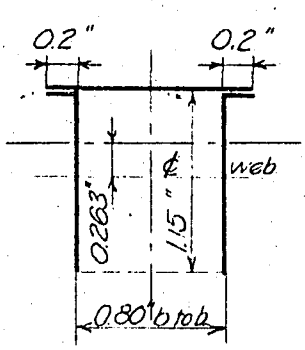
3. Practical Considerations. The foregoing detailed design has been made on the assumption that the materials used are suitable for model construction and that all practical difficulties could be overcome. A few remarks will be made here for some problems which need special attention.

(a) Problem of Latticing.--To cut thin sheet steel for latticing or bracing different parts of all members by welding or riveting as for actual bridges is entirely impracticable, because welding would burn off those thin pieces. It was finally decided to use open web latticing, that is, for bottom chords a hollow rectangular section is made to meet the desired dimension, and then the top and bottom parts are cut open by a series of triangular holes whose edges make 45 degrees with the edges of the member.

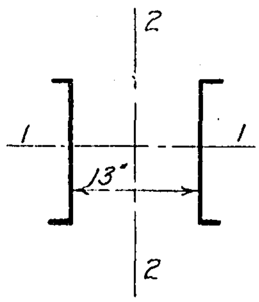
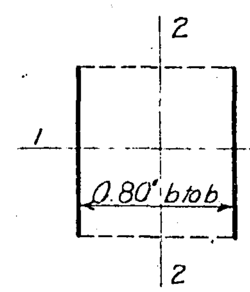
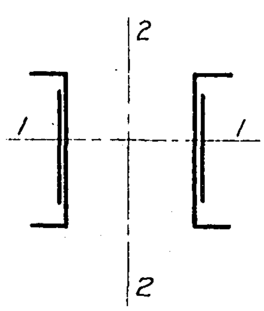
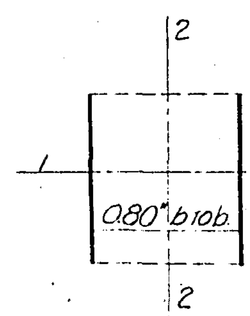
MODEL SECTIONS

Member	Prototype		
	Section	Structural properties	Section
End post	 1 Cov.Pl. 21"x5/8" 2 /s 3½"x3½"x7/16" 2 /s 5"x3½"x7/8" 2 Webs 18"x5/8"	A = 49.45 sq. in. I_{11} = 2611.5 $in.^4$ I_{22} = 3008.6 $in.^4$	 1 Cov.Pl. 1.20"x0.03125 2 Web /s 1.14x0.2312x 0.03125"
Top chord	 1 Cov.Pl. 21"x3/8" 2 /s 3½"x3½"x3/8" 2 Webs 18"x½" 2 /s 5"x3½"x13/16"	A = 43.33 sq. in. I_{11} = 2385.4 $in.^4$ I_{22} = 2425.2 $in.^4$	 1 Cov.Pl. 1.20"x0.03125 2 Web /s 1.15"x0.225"x0

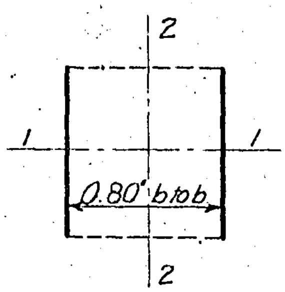
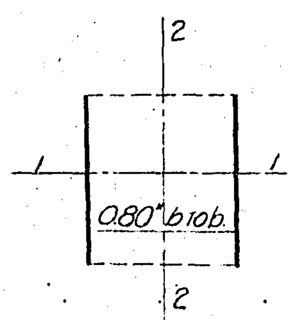
MODEL SECTIONS

	Model			
ructural pperties	Section	Properties: Required	Properties: Furnished	Error %
: 49.45 q. in. . = 2611.5 in^4. : = 3008.6 in^4.	 1 Cov.Pl. 1.20"x0.03125" 2 Web /s 1.14x0.2312x 0.03125"	$\frac{A}{n^2}$ = 0.1236 sq. in. $\frac{I_{11}}{n^4}$ = 0.01632 in^4. $\frac{I_{22}}{n^4}$ = 0.01880 in^4.	A = 0.1213 sq. in. I_{11} = 0.01747 in^4. I_{22} = 0.01818 in^4.	-1.9 +7.0 -3.3
: 43.33 q. in. = 2385.4 in^4. = 2425.2 in^4.	 1 Cov.Pl. 1.20"x0.03125" 2 Web /s 1.15"x0.225"x0.025"	$\frac{A}{n^2}$ = 0.1083 sq. in. $\frac{I_{11}}{n^4}$ = 0.01494 in^4. $\frac{I_{22}}{n^4}$ = 0.01515 in^4.	A = 0.1050 sq. in. I_{11} = 0.01523 in^4. I_{22} = 0.01658 in^4.	-2.0 +1.9 +9.4

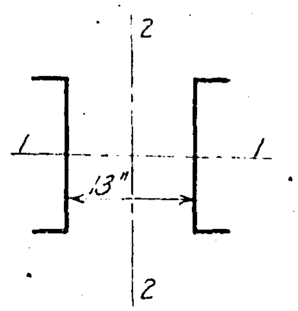
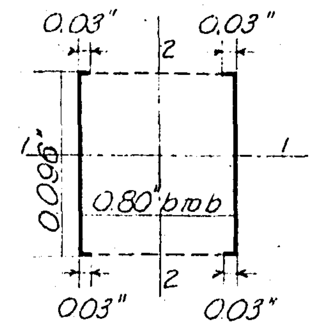
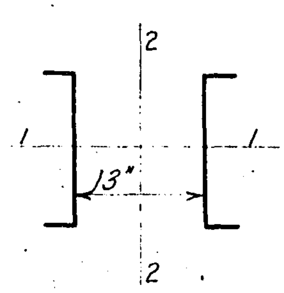
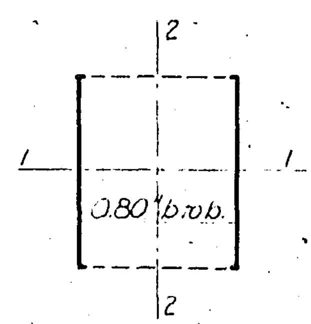
MODEL SECTIONS

Member	Prototype		
	Section	Structural properties	Section
Bottom chords in end panels	 2 15" 45 lb. channels	A = 26.48 sq. in. I_{11} = 750.2 in^4. I_{22} = 1427.0 in^4.	 2 Webs 0.883"x0.03
Bottom chords in mid-panels	 2 15" 55 lb. channels 2 Pls. 12"x$\frac{5}{8}$"	A = 50.4 sq. in. I_{11} = 1076.0 in^4. I_{22} = 2824.0 in^4.	 2 Webs 0.83"x0.07

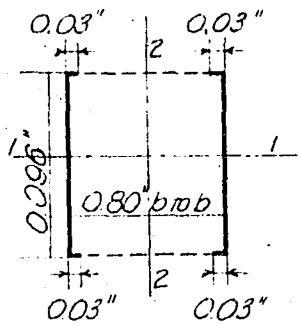
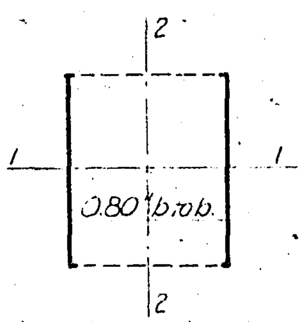
MODEL SECTIONS

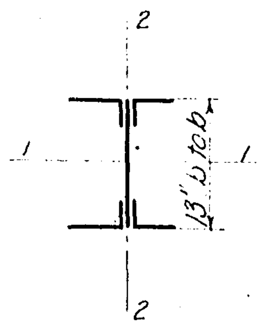
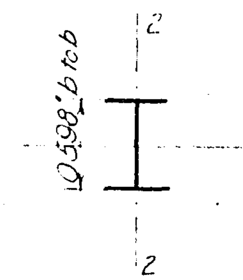
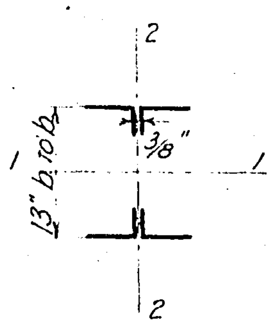
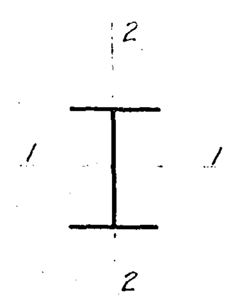
	Model			
ral ies	Section	Properties		Error %
		Required	Furnished	
.48 n. '50.2 .427.0	 2 Webs 0.883"x0.0375"	$\frac{A}{n^2}$ = 0.0662 sq. in. $\frac{I_{11}}{n^4}$ = 0.00468 in^4. $\frac{I_{22}}{n^4}$ = 0.00893 in^4.	A = 0.0662 sq. in. I_{11} = 0.00430 in^4. I_{22} = 0.00961 in^4.	0 +8.1 +7.6
.4 1. 1076.0 4 2824.0 4	 2 Webs 0.83"x0.0748"	$\frac{A}{n^2}$ = 0.126 sq. in. $\frac{I_{11}}{n^4}$ = 0.006725 in^4. $\frac{I_{22}}{n^4}$ = 0.01765 in^4.	A = 0.124 sq. in. I_{11} = 0.00710 in^4. I_{22} = 0.01638 in^4.	-1.6 +5.5 -7.2

MODEL SECTIONS

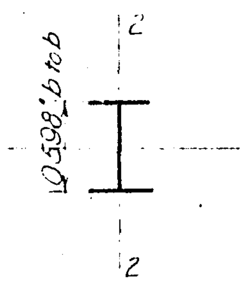
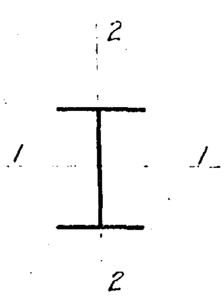
Member	Prototype		
	Section	Structural properties	Section
Diagonals U_1- L_2 U_5- L_4 in panels 2 & 5	 2 15" 50 lb. channels	A = 29.42 sq. in. I_{11} = 805.4 in^4. I_{22} = 1582.7 in^4.	 2 Webs 0.906"x0.0375" w 0.03" bent
Diagonals U_3- L_2 U_3- L_4 in panels 3 & 4	 2 15" 55 lb. channels	A = 32.36 sq. in. I_{11} = 860.4 in^4. I_{22} = 1750.4 in^4.	 2 Webs 0.98"x0.0375".

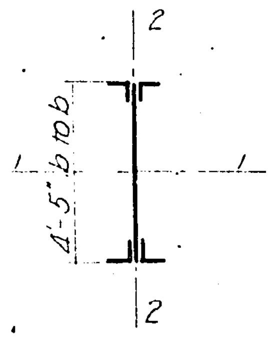
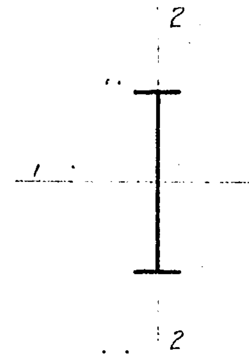
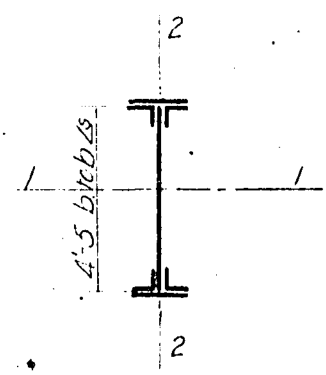
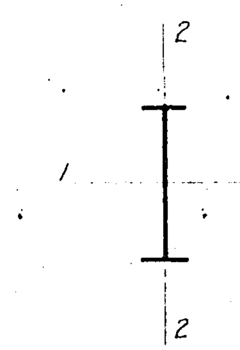
MODEL SECTIONS

tural rties	Model Section	Properties Required	 Furnished	 Error %
9.42 in. 805.4 n^4. 1582.7 n^4.	 2 Webs 0.906"x0.0375" with 0.03" bent	$\frac{A}{n^2}$ = 0.07355 sq. in. $\frac{I_{11}}{n^4}$ = 0.005034 in^4. $\frac{I_{22}}{n^4}$ = 0.009891 in^4.	A = 0.073 sq. in. I_{11} = 0.0054 in^4. I_{22} = 0.0105 in^4.	-0.6 + 5 + 4
52.36 in. = 860.4 in^4. = 1750.4 in^4.	 2 Webs 0.98"x0.0375"	$\frac{A}{n^2}$ = 0.0809 sq. in. $\frac{I_{11}}{n^4}$ = 0.0054 in^4. $\frac{I_{22}}{n^4}$ = 0.01094 in^4.	A = 0.0735 sq. in. I_{11} = 0.00586 in^4. I_{22} = 0.01067 in^4.	-9.1 +6.7 -2.4

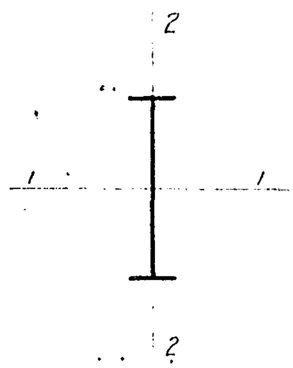
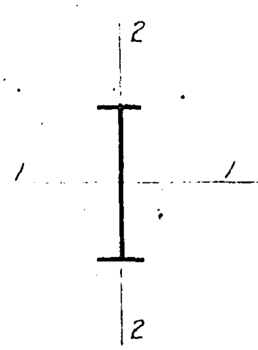
Member	Prototype		
	Section	Structural properties	Section
Verticals U_1-L_1 U_3-L_3 U_5-L_5	 4 ∠s 5"x3$\frac{1}{2}$"x$^9/16$" 1 Web 11$\frac{1}{2}$"x$^9/16$"	A = 24.35 sq. in. I_{11} = 559.9 in^4. I_{22} = 112.86 in^4.	 2 Pls. 0.49"x0.0364' 1 Web 0.525"x0.0487' 2 Fillers 0.10"
Verticals U_2-L_2 U_4-L_4	 4 ∠s 5"x3$\frac{1}{2}$"x$^3/8$" Webs at ends	A = 12.20 sq. in. I_{11} = 343.04 in^4. I_{22} = 70.62 in^4.	 2 Pls. 0.52"x0.0187 1 Web 0.606"x0.0187 2 Fillers 0.0748"

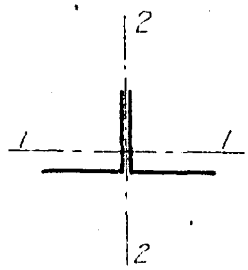
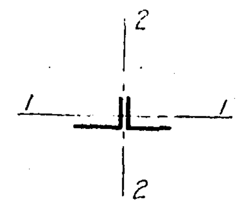
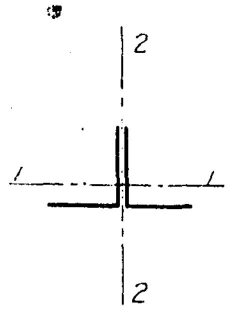
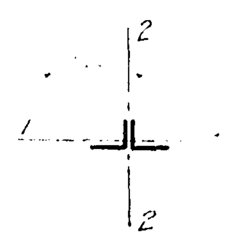
MODEL SECTIONS

	Model			
l s	Section	Properties		Error %
		Required	Furnished	
5 .).9 2.86	 2 Pls. 0.49"x0.0364" 1 Web 0.525"x0.0487" 2 Fillers 0.10"	$\frac{A}{n^2}$ = 0.06088 sq. in. $\frac{I_{11}}{n^4}$ = 0.003499 $in.^4$ $\frac{I_{22}}{n^4}$ = 0.000705 in^4.	A = 0.06125 sq. in. I_{11} = 0.00341 $in.^4$ I_{22} = 0.000713 in^4.	+0.7 -2.6 +1.2
0 1. 13.04).62	 2 Pls. 0.52"x0.01875" 1 Web 0.606"x0.01875" 2 Fillers 0.0748"	$\frac{A}{n^2}$ = 0.0305 sq. in. $\frac{I_{11}}{n^4}$ = 0.002144 $in.^4$ $\frac{I_{22}}{n^4}$ =0.0004414 $in.^4$	A = 0.0308 sq. in. I_{11} = 0.002248 $in.^4$ I_{22} =0.000439 $in.^4$	+1.2 +4.8 -0.8

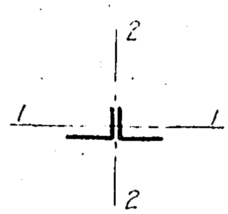
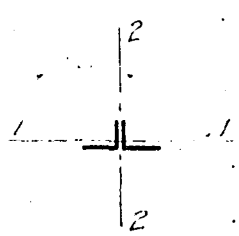
Member	Prototype		
	Section	Structural properties	Section
End floor beam	 4 $\angle$s 6"x6"x$^9/_{16}$" 1 Web pl. 52$\frac{1}{2}$"x$^3/_8$"	A = 45.40 sq. in. I_{11} = 20416.47 in^4. I_{22} = 181.47 in^4.	 2 Pls. 0.65"x0.02 1 Web 2.8"x0.0312
Inter-ior floor beam	 4 $\angle$s 6"x6"x$\frac{1}{2}$" 1 Web 52$\frac{1}{2}$"x$^7/_{16}$" 2 Cov. Pls. 12"x$^7/_{16}$"	A = 56.47 sq. in. I_{11} = 27020 in^4. I_{22} = 286.82 in^4.	 2 Pls. 0.705"x0. 1 Web 2.975"x0.0

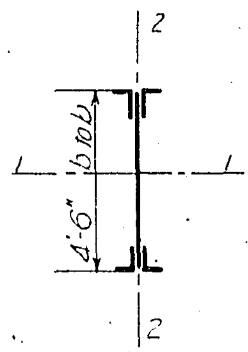
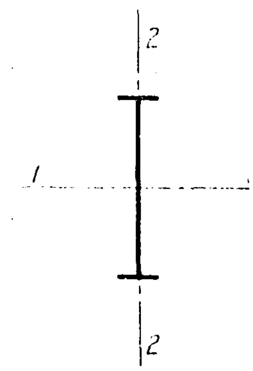
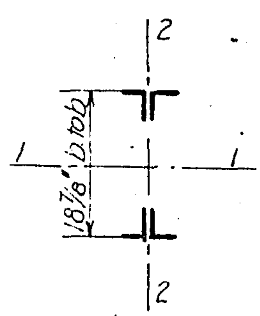
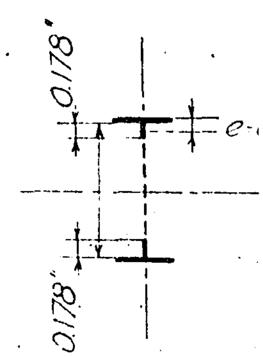
MODEL SECTIONS

	Model			
ctural erties	Section	Properties: Required	Properties: Furnished	Error %
45.40 . in. = 20416.47 in.^{4} = 181.47 in.^{4}	 2 Pls. 0.65"x0.025" 1 Web 2.8"x0.03125"	$\frac{A}{n^2}$ = 0.1135 sq. in. $\frac{I_{11}}{n^4}$ = 0.1276 in.^{4} $\frac{I_{22}}{n^4}$ =0.001134 in.^{4}	A = 0.1200 sq. in. I_{11} = 0.1222 in.^{4} I_{22} = 0.001144 in.^{4}	+5.7 -4.2 +0.9
56.47 q. in. = 27020 in.^{4} = 286.82 in.^{4}	 2 Pls. 0.705"x0.03125" 1 Web 2.975"x0.03125"	$\frac{A}{n^2}$ = 0.1412 sq. in. $\frac{I_{11}}{n^4}$ = 0.1689 in.^{4} $\frac{I_{22}}{n^4}$ = 0.001792 in.^{4}	A = 0.1377 sq. in. I_{11} = 0.1708 in.^{4} I_{22} = 0.00181 in.^{4}.	-2.4 +1.0 +2.0

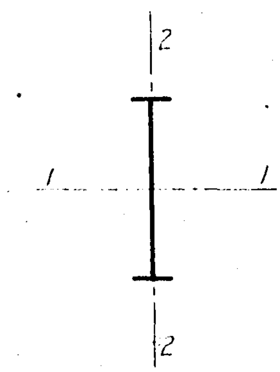
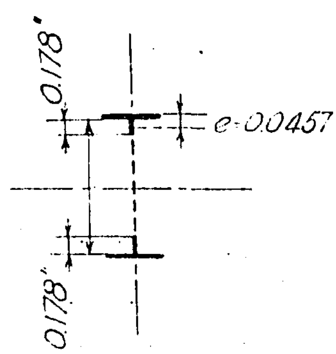
Member	Prototype		MODEL SECTIONS
	Section	Structural properties	Section
Bottom Lateral Diagonals in end panels	 2 /s 3"x3"x7/16"	$A = 4.86$ sq. in. $I_{11} = 4.00$ in^4. $I_{22} = 8.02$ in^4.	 2 /s bent from pls. 0.163"x0.158"x0.018
Bottom Lateral Diagonals in interior panels	 2 /s 3"x3"x3/8"	$A = 4.22$ sq. in. $I_{11} = 3.60$ in^4. $I_{22} = 6.94$ in^4.	 2 /s bent from pls. 0.15"x0.15"x0.01875"

MODEL SECTIONS

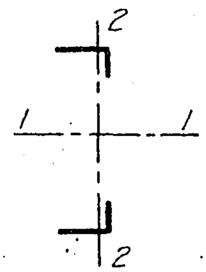
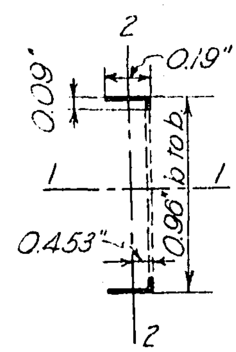
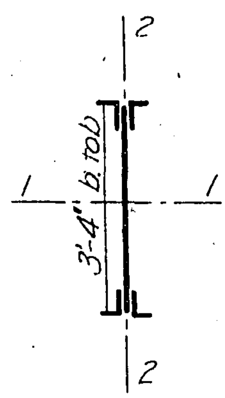
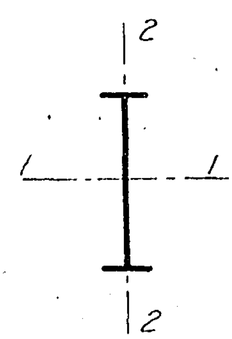
	Model			
...ural ...ties	Section	Properties: Required	Properties: Furnished	Error %
; i.		$\frac{A}{n^2}$ = 0.01215 sq. in.	A = 0.01135 sq. in.	-6.5
.00		$\frac{I_{11}}{n^4} = 0.25 \times 10^{-4}$ in^4.	$I_{11} = 0.263 \times 10^{-4}$ in^4.	+5.1
.02	2 ∠s bent from pls. 0.163"x0.158"x0.01875"	$\frac{I_{22}}{n^4} = 0.50 \times 10^{-4}$ in^4.	$I_{22} = 0.54 \times 10^{-4}$ in^4.	+8.0
22 in.		$\frac{A}{n^2}$ = 0.01055 sq. in.	A = 0.01055	0
3.60 4		$\frac{I_{11}}{n^4} = 0.225 \times 10^{-4}$ in^4.	$I_{11} = 0.225 \times 10^{-4}$	0
6.94 4	2 ∠s bent from pls. 0.15"x0.15"x0.01875"	$\frac{I_{22}}{n^4} = 0.434 \times 10^{-4}$ in^4.	$I_{22} = 0.434 \times 10^{-4}$	0

Member	Prototype		
	Section	Structural properties	Section
Portal Struts	 4 $\angle$s 3"x3"x$^3/_8$" 1 Web 53½"x$^3/_8$"	A = 28.50 sq. in. I_{11} = 10546.3 in.^{4} I_{22} = 17.20 in.^{4}	 2 Pls. 0.3"x0.02 1 Web 2.85"x0.01
Head Struts	 4 $\angle$s 3"x3"x$^5/_{16}$" with 2½"x$^3/_8$" Lattice	A = 7.12 sq. in. I_{11} = 528.8 in.^{4} I_{22} = 14.0 in.^{4}	 2 Flange Pls. 0.304"x0.01875 1 Web 0.904"x0.0 Solid at ends

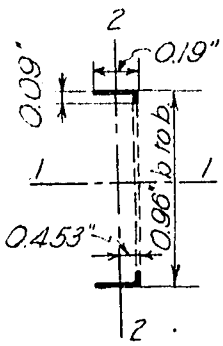
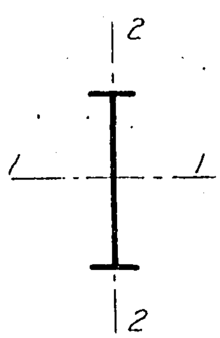
MODEL SECTIONS

	Model			
tructural roperties	Section	Properties		Error %
		Required	Furnished	
= 28.50 sq. in. 1 = 10546.3 in^4. 2 = 17.20 in^4.	 2 Pls. 0.3"x0.025" 1 Web 2.85"x0.01875"	$\frac{A}{n^2}$ = 0.07125 sq. in. $\frac{I_{11}}{n^4}$ = 0.0659 in^4 $\frac{I_{22}}{n^4}$ =0.0001075 in^4.	A = 0.0714 sq. in. I_{11} = 0.0663 in^4. I_{22} = 0.000108 in^4.	+0.2 +0.7 +0.6
= 7.12 q. in. 1 = 528.8 in^4. 22 = 14.0 in^4.	 2 Flange Pls. 0.304"x0.01875" 1 Web 0.904"x0.01875" Solid at ends	$\frac{A}{n^2}$ = 0.0178 sq. in. $\frac{I_{11}}{n^4}$ = 0.003305 in^4. $\frac{I_{22}}{n^4}$ =0.875x10^{-4} in^4.	A = 0.01807 sq. in. I_{11} = 0.003305 in^4. I_{22} =0.0000878 in^4.	+1.2 0 +0.2

MODEL SECTIONS

Member	Prototype Section	Prototype Structural properties	Section
Top Lateral Diago-nals	 2 /s 3"x4"x$^5/_{16}$" 18$^7/_8$" b. to. b.	A = 4.18 sq. in. I_{11} = 318.3 in^4. I_{22} = 6.8 in^4.	 0.01875" Pl. bent. with cut web Lattice
String-ers	 4 /s 6"x6"x$^7/_8$" 1 Web 39½"x$^3/_8$"	A = 53.73 sq. in. I_{11}=141917.05 in^4. I_{22} = 285.02 in^4.	 2 Pls. 0.66"x0.0375 1 Web 2.2"x0.0375"

MODEL SECTIONS

ictural / perties	Model			
	Section	Properties		Error %
		Required	Furnished	
.18 in.		$\frac{A}{n^2}$ = 0.01045 sq. in.	A = 0.0105 sq. in.	+0.5
= 318.3 in^4.		$\frac{I_{11}}{n^4}$ = 0.001989 in^4.	I_{11} = 0.00214 in^4.	+6.9
= 6.8 in^4.	0.01875" Pl. bent. with cut web Lattice	$\frac{I_{22}}{n^4}$ = 0.0000425 in^4.	I_{22} = 0.0000407 in^4.	-4.2
53.73 in.		$\frac{A}{n^2}$ = 0.1343 sq. in.	A = 0.1320 sq. in.	-1.6
141917.05 in^4.		$\frac{I_{11}}{n^4}$ = 0.0939 in^4.	I_{11} = 0.0946 in^4.	+0.7
= 285.02 in^4.	2 Pls. 0.66"x0.0375" 1 Web 2.2"x0.0375"	$\frac{I_{22}}{n^4}$ = 0.001781 in^4.	I_{22} = 0.00178 in^4.	0

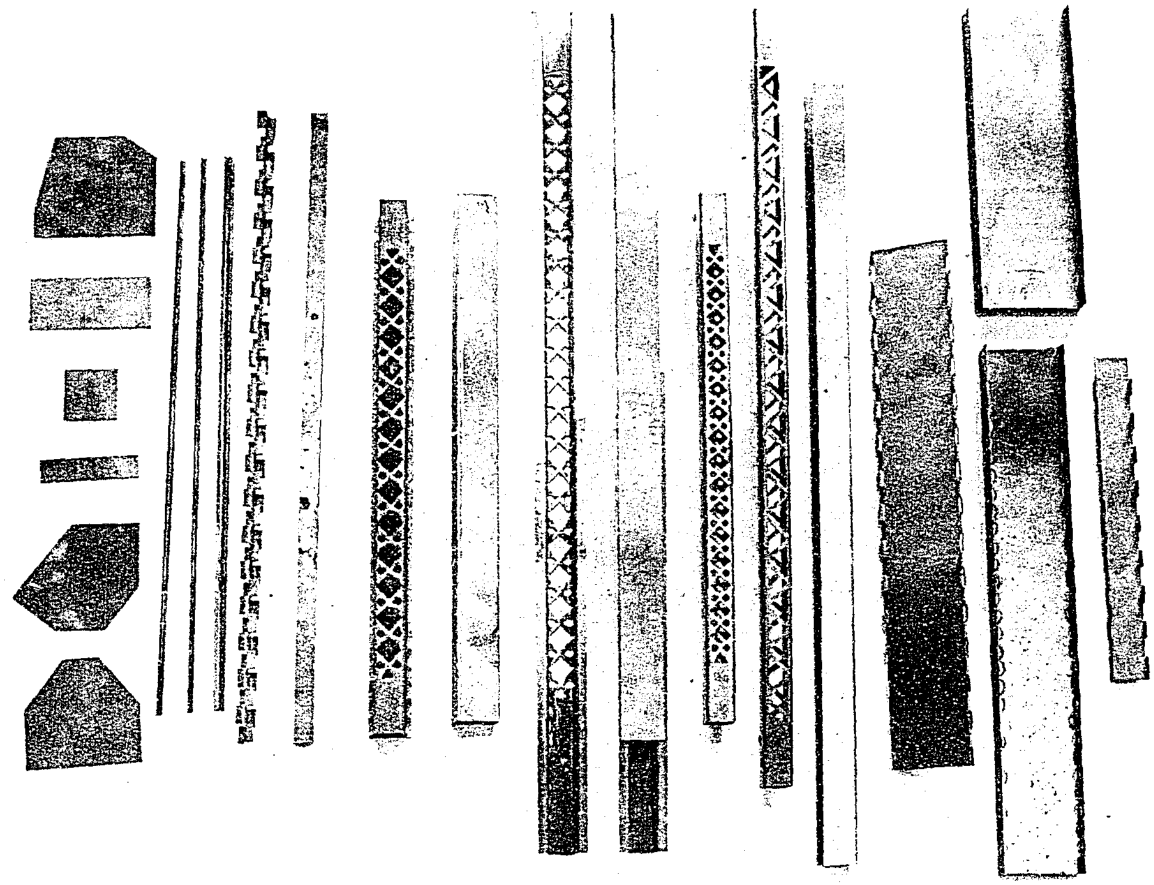


PLATE VII.

TYPICAL MEMBERS OF BRIDGE MODEL

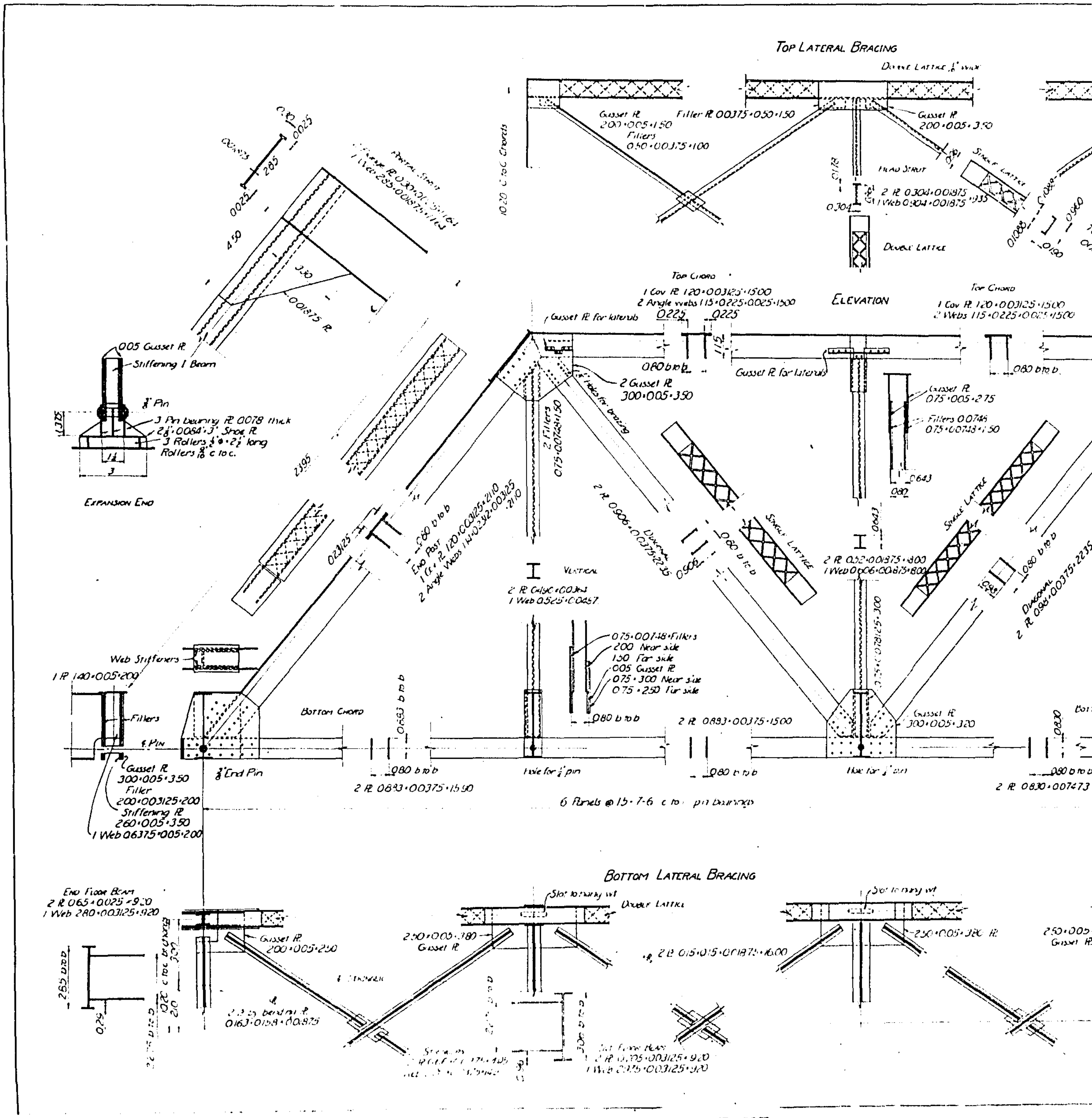


PLATE VI.

DETAILED ASSEMBLY OF BRIDGE MODEL

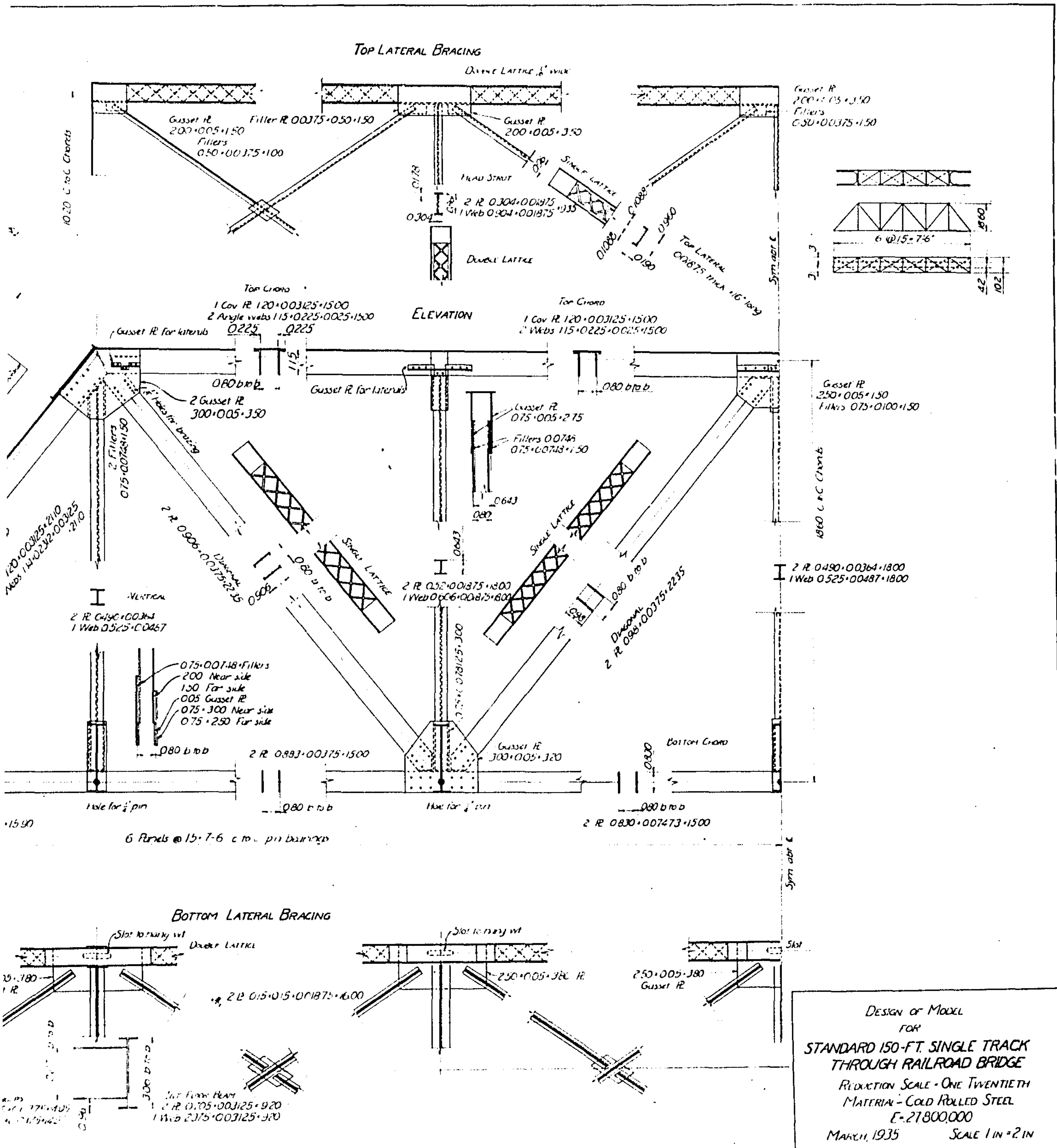


PLATE VI.

DETAILED ASSEMBLY OF BRIDGE MODEL

The width of material left between the open cut parts is made 3/64 of an inch. For single latticing, as some members would require, the diagonals in the open cut portion make an angle at 60 degrees with the edges. This construction makes all theoretical bars of the same thickness as the bridge member itself. Especially for thin members such as head struts and top lateral diagonals, it is very practical and later tests show that structural behavior of members tied together in this way works effectively.

(b) I-beam sections.--Quite a few I-beam sections are necessary for floor beams, stringers, verticals, and portal struts. All I-beam sections have been made with the web portion 3/8 of an inch wider than the theoretical width, and 3/16 of an inch on each side is cut at 1/2 inch intervals and perpendicular to the web plane. These strips are then bent alternately to each side as shown in Plate (VII). Bent strips are finally brazed to the flange plates, edges wider than the flange plates are then sheared off.

For head struts, a solid I-beam section is thus made first, and the necessary lattice is then accomplished by cutting open the required web portion.

(c) Gusset plates and fillers.--The impracticability of fillet welding very thin sheets in assembling the trusses has made it necessary to use gusset plates. All gusset plates are 0.05 inch thick, and a series of 1/8-inch holes were first drilled so that all members could be brazed together at the joints.

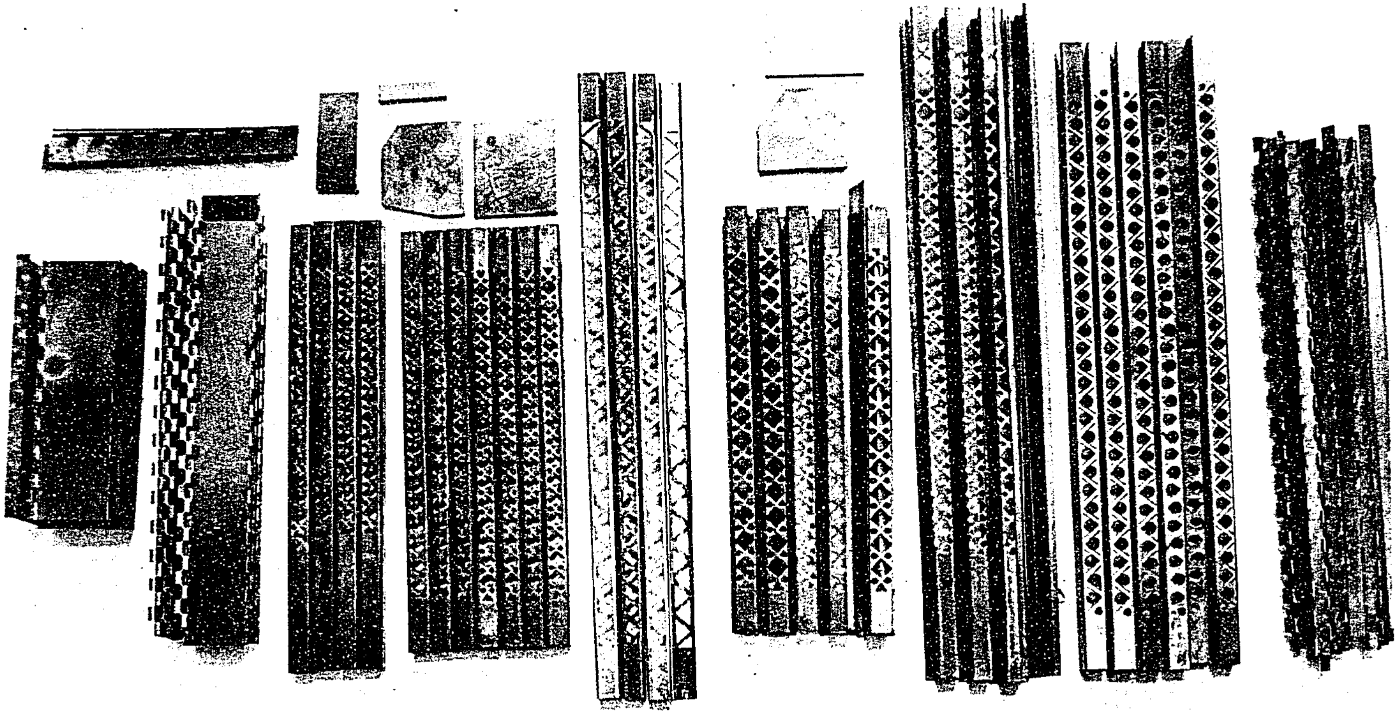


PLATE VIII.

UNASSEMBLED MEMBERS OF BRIDGE MODEL

All bottom chords have an outside width of 0.8 of an inch, and top chords are designed to have the same width by using a cover plate. Main diagonals are designed with a width 0.8 of an inch, and filler plates are brazed to verticals before assembling the main trusses.

Gusset plates for top lateral systems are simply brazed to top chord cover plates for the upper side, and for the lower side, it is found necessary to bend them into angles first and one leg is then brazed to gusset plates of main trusses through drilled holes.

(d) End bearings and loading device.--3/8 of an inch pins are used for the four end bearings, and stiffening I-beams are used at the ends in order to give a better connection for the end floor beams.

At the expansion ends, 1/2 inch rollers are used, and small parallelograms of slender bars are pinned to the rollers to secure motion as a whole. Enough clearances are left between the cylindrical rollers, so that no cutting of rollers as for actual bridges is necessary.

At each panel point, a 1/4 inch hole has been drilled through the joint and a hook is then hung down through the bottom lateral gusset plate. Both the panel pin and the bearing of the joint are designed for an ultimate load of 800 lbs. Since the theory of similitude requires the model to have a density equal to 20 times that of the prototype, any correction of density could be made by loading the panel pins if necessary.

(e) Portal bracing.--A thin sheet of steel having dimensions proportional to the prototype has also been brazed to each top corner of the portal frames. The relative thinness of the knee brace in comparison with top chord cover plates caused a little buckle to occur in the brazing. For further details, see Plate II.

D. STRESS AND DISPLACEMENT MEASUREMENTS ON THE MODEL

1. Preliminary Tests. (a) Modulus of elasticity.--A few specimens have been made for tension tests to determine the elastic behavior of the material to be used. The following results are obtained by tension tests of two specimens, with strains recorded by Huggenberger Tensometers, which read to 1/10000 of an inch per linear inch of strain.

Stress-strain curves are plotted from the data of Table XI, and curves are as shown in Fig. (42). It was found from the table that the average modulus of elasticity of the material is in the neighborhood of 27,000,000 lbs. per sq. in. This value was later used for checking results.

(b) Strength of brazed joints.--Because of the thinness of the materials, spot welding has been abandoned for welding together individual members as well as truss joints.

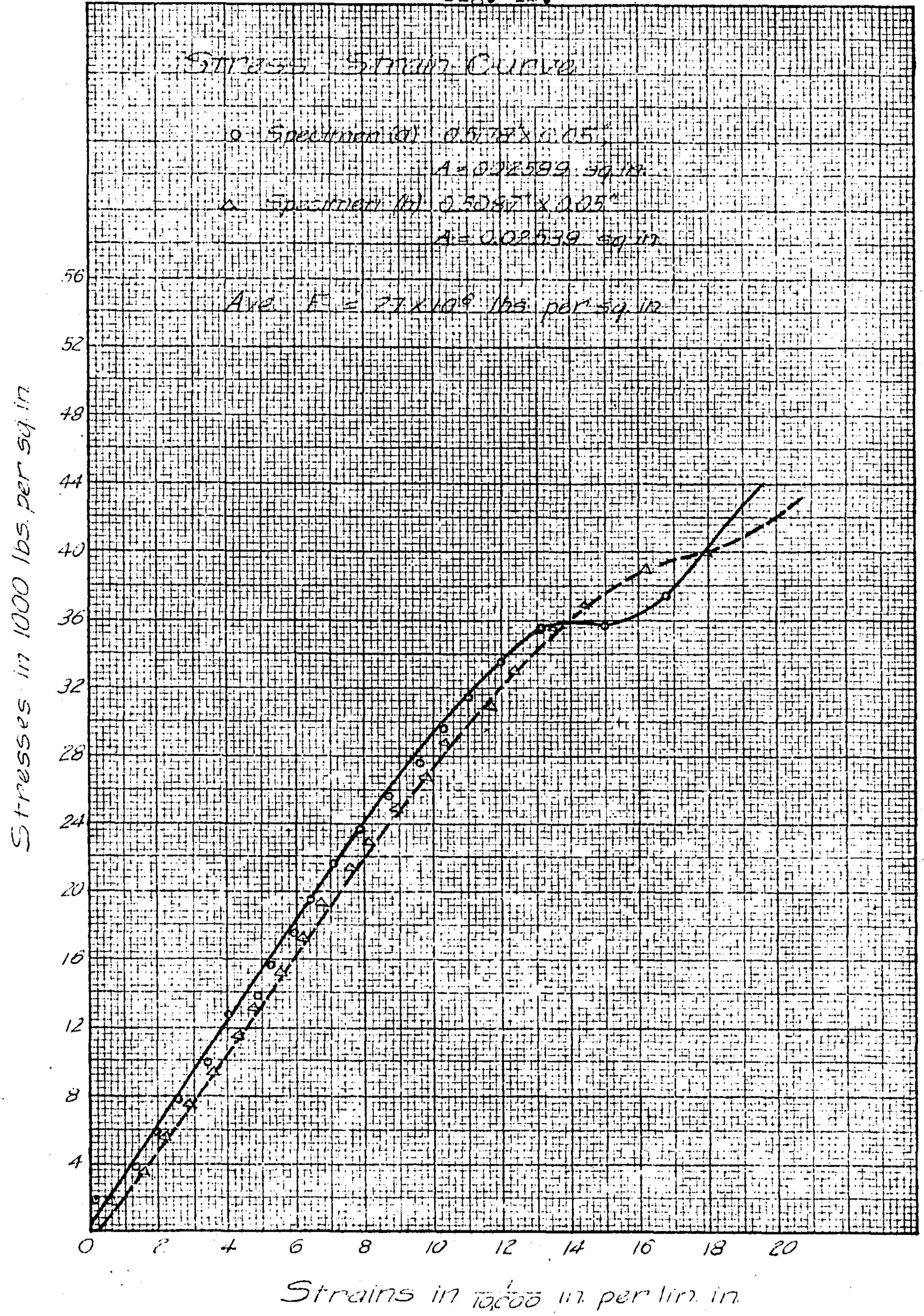
Several series of tests have been made to determine the strength of brazed joints, and to learn whether brazing could be satisfactorily employed for assembling the structure.

TABLE XI. STRESSES AND STRAINS

Load, lbs.	Specimen A		Specimen B	
	Unit stress, lbs./sq. in.	Strain, $\frac{1}{10000}$ in.	Unit stress, lbs./sq. in.	Strain, $\frac{1}{10000}$ in.
50	1970	0.163	1930	0.448
100	3940	1.342	3860	1.225
150	5910	1.915	5790	2.120
200	7880	2.370	7620	2.811
250	9850	3.38	9560	3.505
300	12820	4.035	11580	4.280
350	13770	4.850	13530	4.770
400	15750	5.220	15450	5.540
450	17730	5.870	17370	6.075
500	19690	6.440	19300	6.765
550	21700	7.135	21300	7.540
600	23600	7.865	23200	8.030
650	25600	8.725	25100	8.925
700	27500	9.580	27000	9.665
750	29500	10.315	28900	10.235
800	31500	11.045	30900	11.705
850	33500	12.065	32800	12.310
900	35500	13.135	35700	13.215
950	37400	15.045	36700	14.440
1000	39400	16.840	38600	16.110
1040	40900	Yielding		
1050	41300		40600	Yielding
1295	51000		50000	Ultimate
1340	52800	Ultimate		

Form E-5

Fig. 42.



Results of the tension tests have shown that out of eight fillet welded specimens, seven failed in the body of the members and one failed in the joint. Similar results were obtained for tests conducted for six members or specimens which were brazed together in different ways as through cut slots or drilled holes; all of them failed in tension.

It was decided then to use the brazing method to fasten the cover plate to the webs in top chords, as well as in all the I-beam sections. For assembling of the trusses, a series of holes have been drilled through the gusset plates and then later filled with brass.

(c) Column tests of top chords.--A sample top chord section was built in the specified way, that is, cover plate brazed to the bent web edges and the web then cut open at the bottom leaving 3/64 inch diagonal to form the desired lattice bracings.

Results of the tests are tabulated as shown and a series of curves plotted to determine the top chord behavior. It was observed that no buckling of the web or any failure of the brazing occurred. The top chord specimen behaved quite satisfactorily as a compression section.

(d) General behavior of the model.--After the entire model was assembled, a series of tests was made to determine its elastic properties. The first series of tests, however, was conducted with such loadings as to produce results which are known already.

Thus, the model was uniformly loaded with 100 lbs. per panel

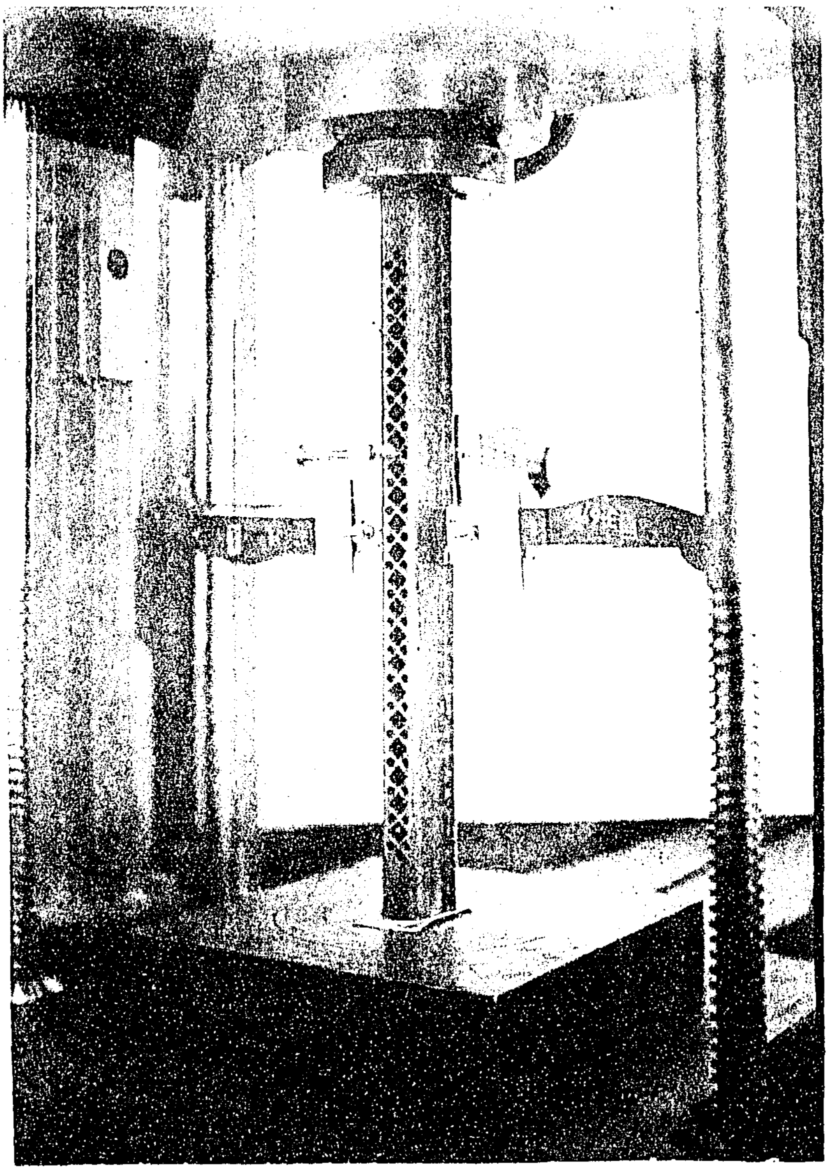


PLATE IX.

COLUMN TEST OF TYPICAL TOP CHORD SECTION

TABLE XII. COLUMN TEST

SET "A"

(1)	(2)	(3) Strains in $\frac{1}{10000}$ in./in.			(4)	(5)
No. of readings	Load, lbs.	# 657 c=1220	# 658 c=1232	# 659 c=1196	Stresses, lbs./sq.in.	Remarks
1	0	0	0	0	0	This set of readings taken when lead pls. yield most.
2	50	0	0.162	0	290	
3	100	0	0.325	0.084	582	
4	150	0.082	0.406	0.167	873	
5	200	0.164	0.487	0.251	1160	
6	300	0.410	0.731	0.334	1740	
7	350	0.410	0.812	0.502	2030	
8	400	0.492	0.823	0.502	2330	
9	450	0.574	1.060	0.669	2610	
10	500	0.656	1.220	0.753	2910	
11	550	0.820	1.380	0.836	3200	
12	600	0.820	1.460	0.920	3490	
13	650	0.902	1.540	1.000	3780	
14	700	1.065	1.620	1.000	4070	
15	750	1.230	1.700	1.090	4360	
16	800	1.230	1.790	1.170	4650	
17	900	1.640	1.870	1.420	5230	
18	1000	1.800	2.030	1.590	5820	
19	1100	2.050	2.270	1.760	6400	
20	1200	2.210	2.520	1.920	6980	
21	1300	2.460	2.760	2.010	7560	
22	1400	2.460	3.000	2.090	8140	
23	1500	2.620	3.250	2.260	8730	
24	1600	2.870	3.410	2.420	9310	
25	1700	3.030	3.650	2.590	9890	
26	1800	3.200	3.900	2.760	10460	
27	1900	3.360	4.060	2.840	11040	
28	2000	3.520	4.300	3.010	11620	

TABLE XII. (CONT.) COLUMN TEST

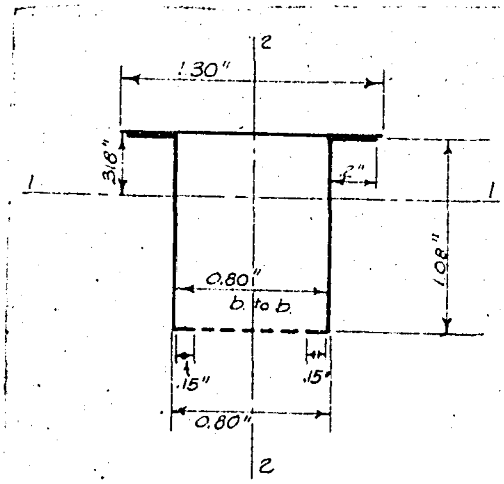
SET "B"

(1) Load, lbs.	(2) Stresses lbs./sq.in.	(3) Strains in $\frac{1}{10000}$ in./in. # 657	# 658	# 659	(4) Remarks
0	0	0.082	0.081	0.084	
50	290	0.082	0.244	0.084	Readings are taken with decreasing load continued from set "A"
100	582	0.164	0.406	0.084	
200	1160	0.246	0.649	0.084	
300	1740	0.410	0.823	0.167	
400	2330	0.574	0.823	0.251	
500	2910	0.738	1.220	0.334	
600	3490	0.902	1.460	0.418	
700	4070	1.065	1.620	0.585	
800	4650	1.230	1.870	0.753	
900	5230	1.480	2.030	0.920	
1000	5820	1.640	2.270	1.090	
1100	6400	1.800	2.520	1.250	
1200	6980	1.970	2.680	1.420	
1300	7560	2.130	2.840	1.590	
1400	8140	2.460	3.080	1.840	
1500	8730	2.540	3.250	2.010	
1600	9310	2.790	3.490	2.090	
1700	9890	2.950	3.730	2.340	
1800	10460	3.200	3.980	2.590	
1900	11040	3.280	4.140	2.760	
2000	11620	3.520	4.300	3.010	

TABLE XII. (CONT.) COLUMN TEST

SET "C"

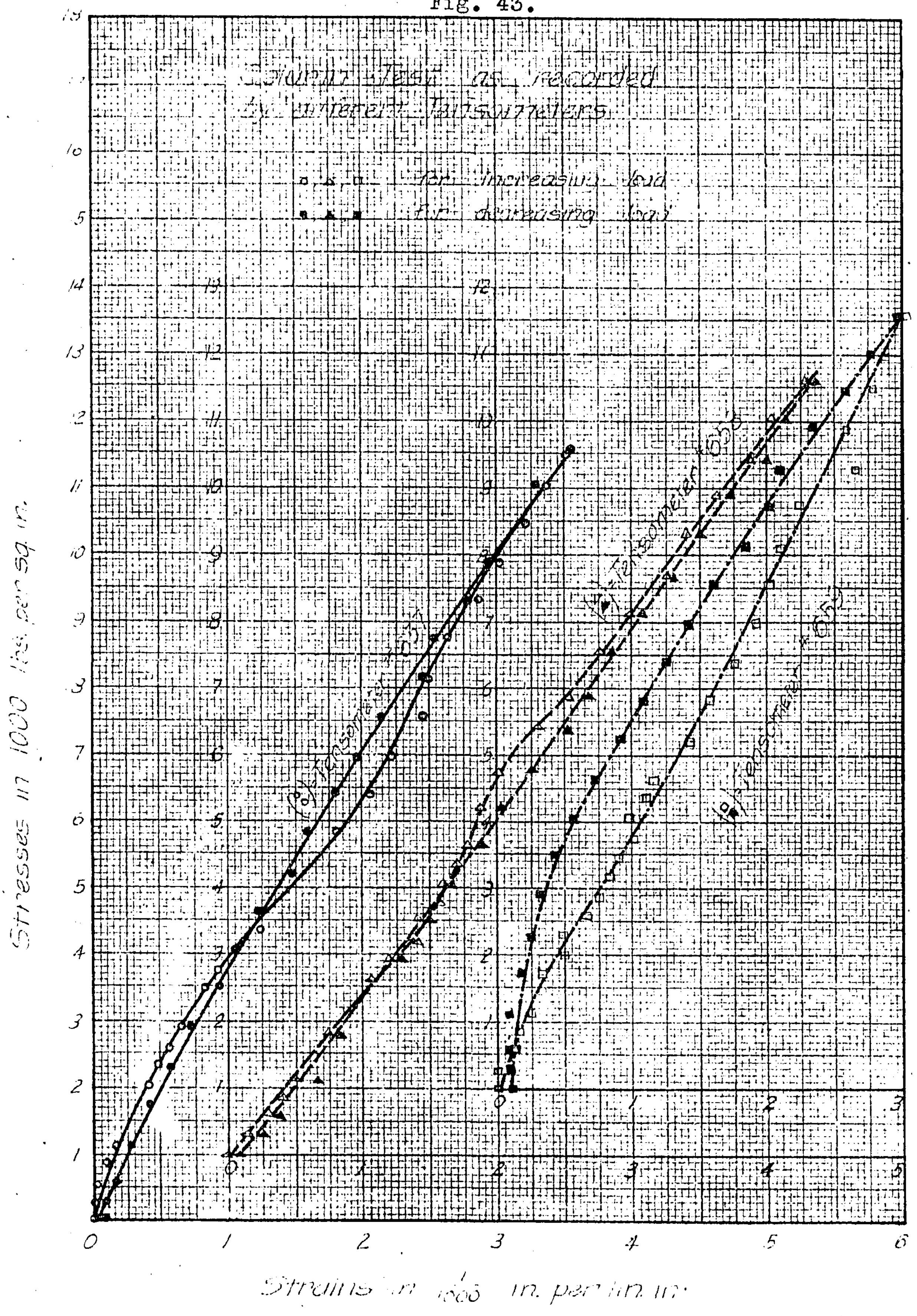
Load, lbs.	Stresses lbs./sq.in.	Strains in $\frac{1}{10000}$ in./in. # 657	# 658	# 659	Strains in $\frac{1}{10000}$ in./in. # 657	# 658	# 659
0	0	0	0	0	0.082	0.162	-0.344
200	1160	0.246	0.568	0.084	0.246	0.731	-0.251
400	2330	0.492	0.974	0.334	0.492	1.220	-0.167
600	3490	0.820	1.380	0.502	0.738	1.620	0.084
800	4650	1.230	1.790	0.836	1.150	2.030	0.334
1000	5820	1.560	2.190	1.170	1.560	2.440	0.669
1200	6980	1.970	2.600	1.590	1.890	2.840	1.000
1400	8140	2.380	3.000	1.920	2.290	3.250	1.340
1600	9310	2.790	3.410	2.260	2.700	3.650	1.670
1800	10460	3.200	3.810	2.590	2.950	3.980	2.010
2000	11620	3.610	4.300	2.760			



Length = 14.96"
Area = 0.1716 in²
I_{11} = 0.0274 in⁴
I_{22} = 0.02611 in⁴
r = 0.390"
$\frac{l}{r}$ = 38.50

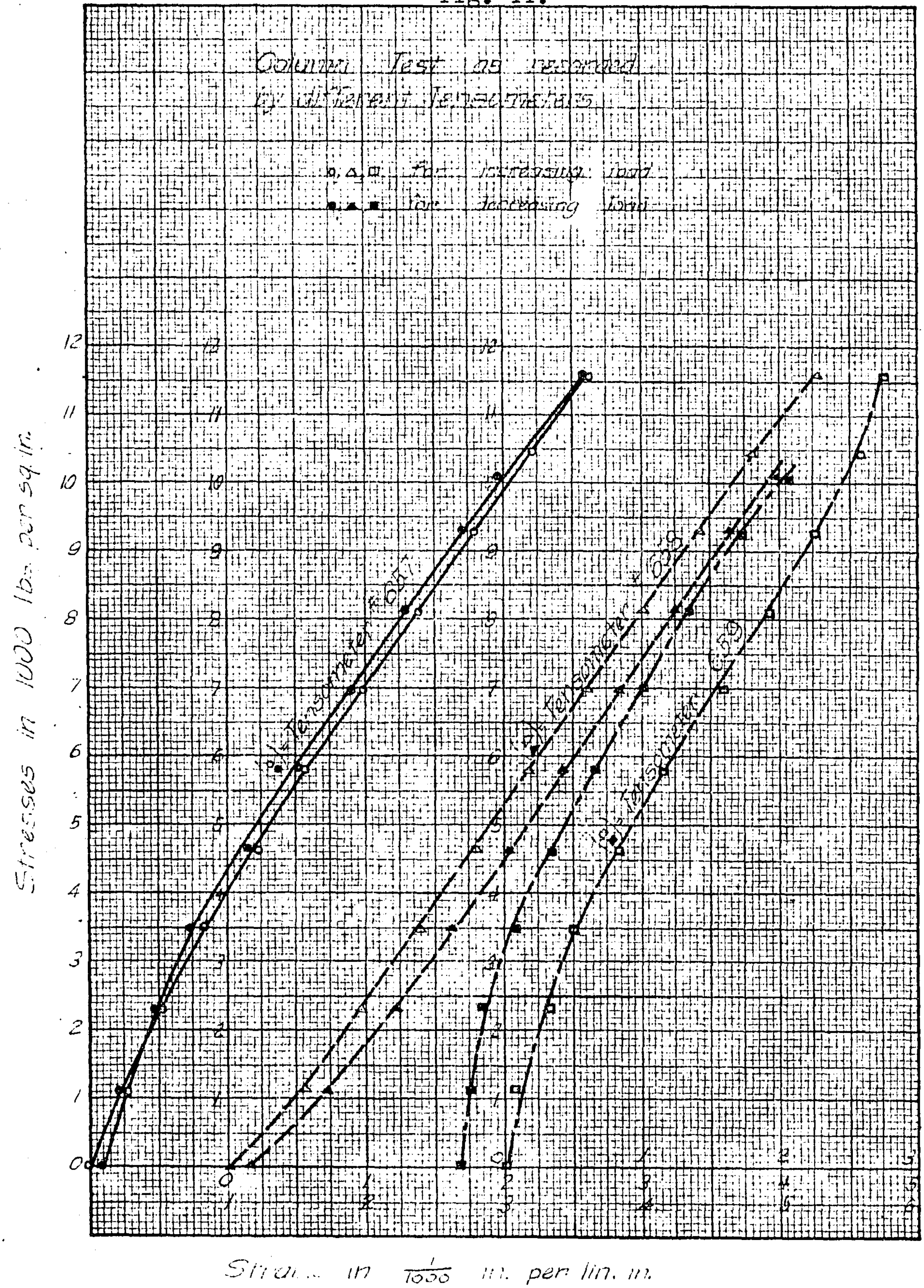
Form E-5

Fig. 43.



Form E-5

Fig. 44.



joint for both trusses, and observations indicated a vertical deflection of 0.0150 of an inch at L_4. For each individual truss taken as a planar structure, the deflection at joint L_4 for 100 lbs. per joint is calculated to be 0.0169 of an inch.

For the same loading, and for the model as a space structure, the deflection would be reduced a little because of the decreased stresses in chord members. An analysis could be easily made by the method outlined in Chapter III as for w_1. Eccentric loadings have not been used in order to avoid the uncertain torsion effect.

The elastic behavior of the model is thus found to be satisfactory. The stresses in a few chord members have also been checked up, and the results agree fairly well with the computed values for uniform loads on planar trusses.

Thus when the model is loaded with 100 lbs. per panel joint for both trusses, the comparison between measured and computed stresses are as follows:

Top chord	Measured stress	Computed stress	Error
U_3-U_4	- 303 lbs.	- 323 lbs.	- 7%
Top chord			
U_{33}-U_{44}	- 334	- 323 lbs.	+ 3%

The computed stresses are made for the model truss considered as a planar structure.

2. Tests to Check the Theoretical Analysis. The most desired results of the model analysis are deflections at the loaded transverse frame, and stresses in a few critical members around the loaded joint. These were accordingly measured.

(a) Deflections.--Dials for measuring deflections at the corners of the loaded transverse frame are located as shown in Fig. (44). Positive deflections will be shown as indicated in Fig. (44b).

In making comparisons between the measured and calculated deflections, calculated values are based upon those obtained in Chapter III - Space Displacements; these values are assumed to vary directly as the load. The computations, however, must first be converted to model dimensions by the corresponding scale reduction factors.

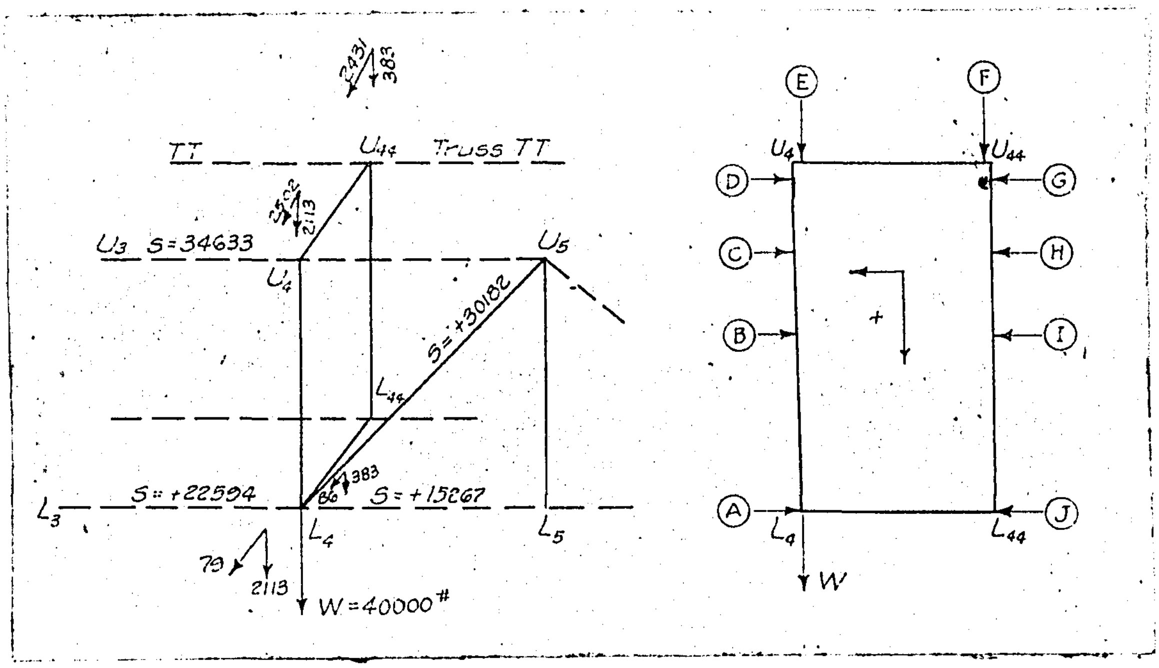
When 40000 lbs. is placed at joint L_4 of the prototype, the computed horizontal deflection at U_4 equals:

$$\frac{2502 \times 1000}{27,000,000} \times 10,000 = 926 \text{ ten thousandths.}$$

This value may be converted to an equivalent model deflection as follows:

$$\frac{2502 \times 1000}{27,000,000} \times \frac{1}{20} \times 10,000 = 46.3 \text{ ten thousandths.}$$

This is the horizontal deflection to expect at U_4 when a 100 lb. load is placed at joint L_4 of the model. For displacements caused by



W = 40000 lbs.

(a) Deflections & Stresses
Deflections in Kips, E=1
Stresses in lbs.

(b) Dial Locations

Fig. 45

Location of Dials

TABLE XIII. DEFLECTIONS OF TRANSVERSE FRAME U_4-U_{44}

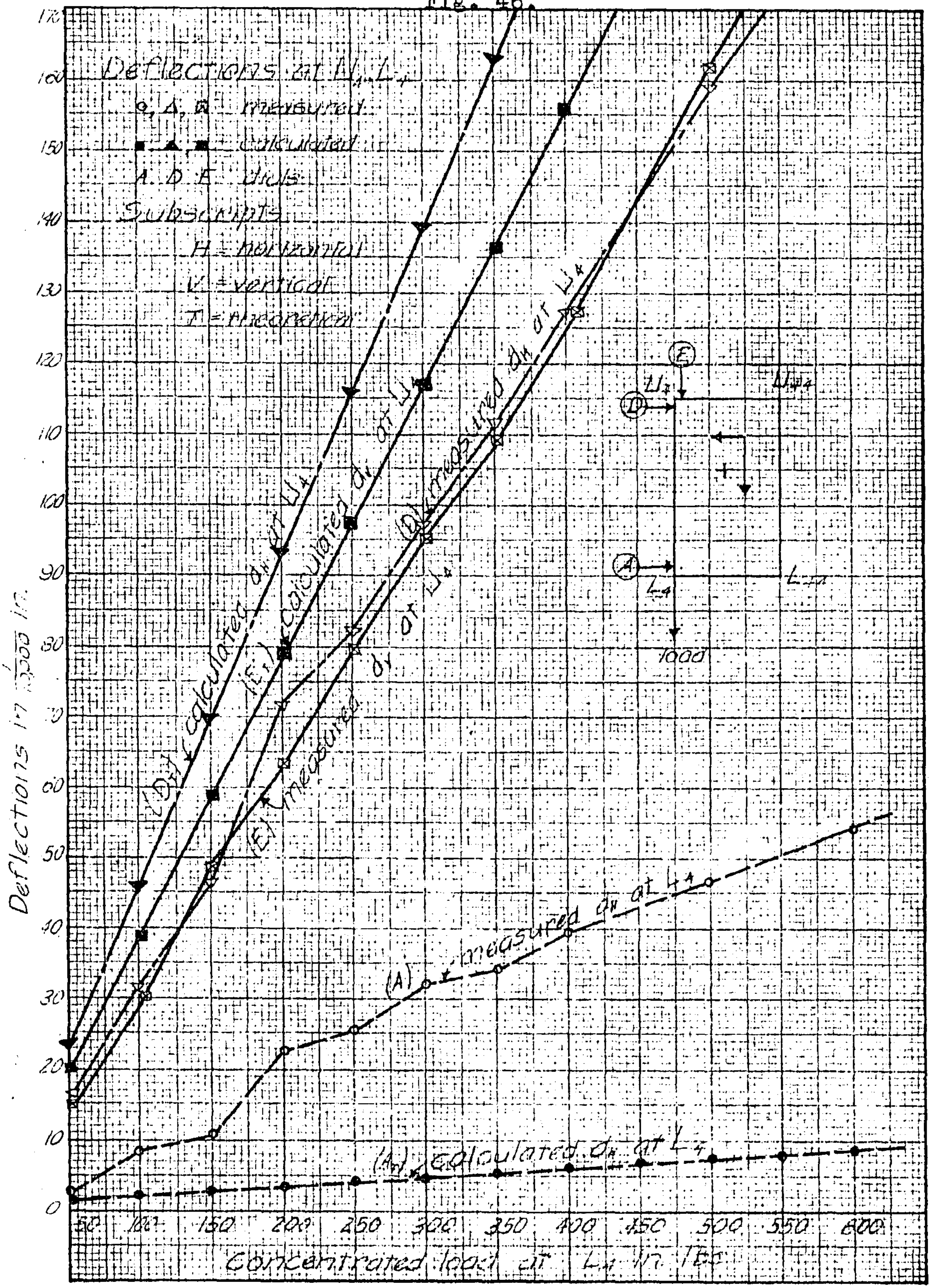
Load at L_4, lbs.	d_H at U_4, horizontal deflection, $\frac{1}{10000}$ in.			d_V at U_4, vertical deflection $\frac{1}{10000}$ in.			d_H at L_4, horizontal deflection, $\frac{1}{10000}$ in.		
	No. of tests in series	Observed value, D	Calculated value	No. of tests in series	Observed value, E	Calculated value	No. of tests in series	Observed value, A	Calculated value
50	10	16.2	23.1	11	15.62	19.6	14	2.57	0.8
100	14	31.2	46.3	14	30.7	39.1	16	8.32	1.5
150	10	46.9	69.5	10	47.5	59	9	10.6	2.2
200	14	71.8	92.5	14	63.5	78	14	23.1	3.0
250	1	82.0	115.	1	80	98	1	25.0	3.7
300	5	97.2	139.	5	95.1	117	1	32.0	4.4
350	2	111.5	159	2	110.5	137	1	34.0	5.1
400	14	127.0	185	14	128.2	156	14	39.3	5.9
500	5	159.0	231	5	162.3	195	5	46.7	7.3
600	13	190.5	277	14	193.5	235	14	54.3	8.8

TABLE XIII. (CONT.) DEFLECTIONS OF TRANSVERSE FRAME U_4-U_{44}

Load at L_4, lbs.	d_H at U_{44}, horizontal deflection, $\frac{1}{10000}$ in.			d_V at U_{44}, vertical deflection, $\frac{1}{10000}$ in.			d_H at L_{44}, horizontal deflection, $\frac{1}{10000}$ in.		
	No. of tests in series	Observed value, G	Calculated value	No. of tests in series	Observed value, F	Calculated value	No. of tests in series	Observed value, J	Calculated value
50	10	14.4	22.5	10	1.4	3.5	11	2.36	0.8
100	6	29.0	45.0	14	3.57	7.1	7	6.50	1.6
150	11	39.0	67.5	10	6.2	10.5	9	10.1	2.4
200	11	56.0	90.0	12	9.0	14.2	9	17.0	3.2
250	1	72.0	112	1	9.0	17.7			
300	4	82.8	135	3	11.33	21.3	3	24.7	4.8
350	2	99.0	157	2	14.0	24.8	--	--	5.6
400	6	116.9	180	12	16.6	28.4	10	30.3	6.4
500	3	143.0	225	3	19.3	35.5	3	37.3	8.0
600	6	169.0	270	12	24.9	42.5	8	45.5	9.6

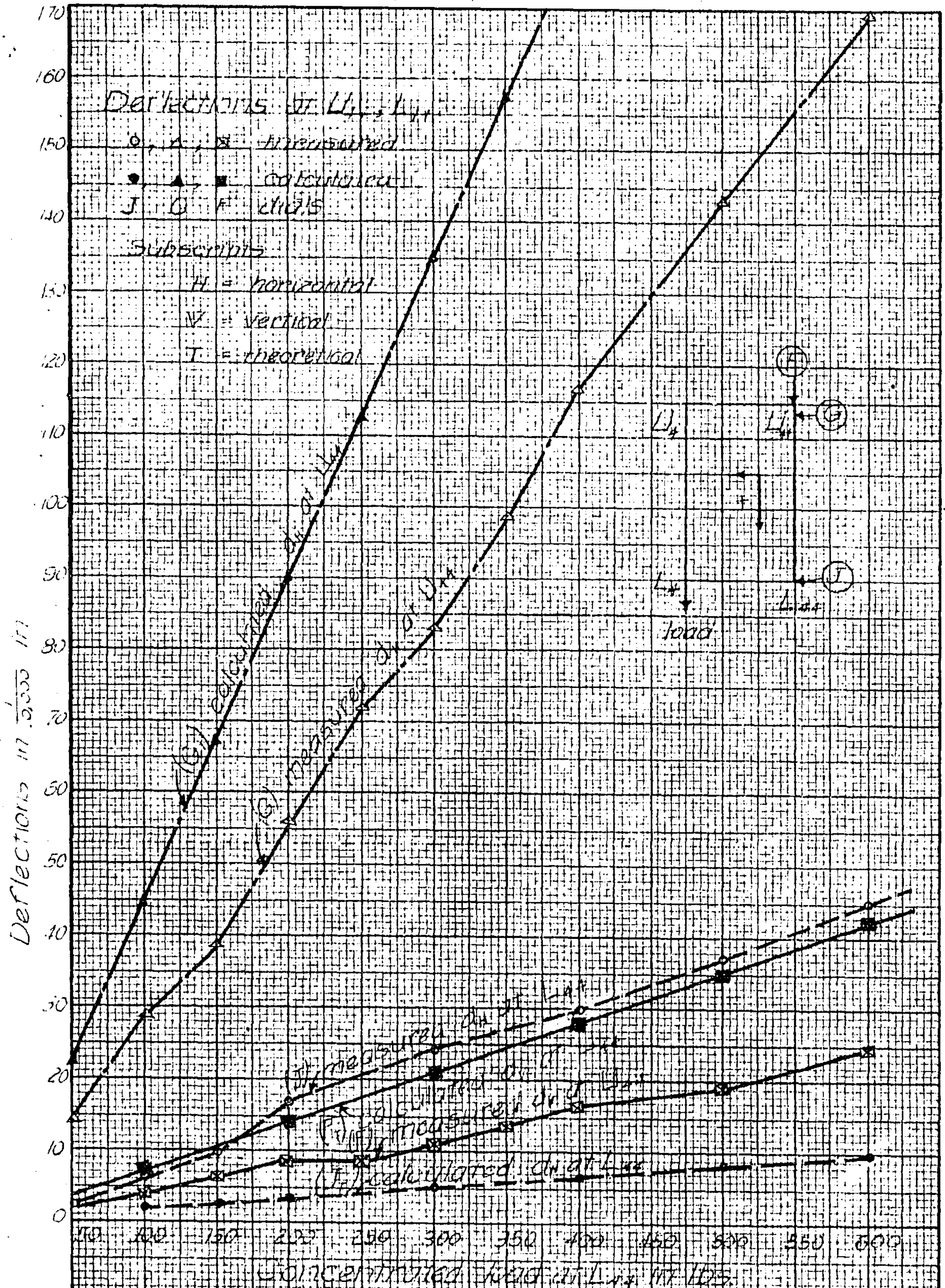
Form E-5

Fig. 46.



Form E-5

Fig. 47.



other loads at L_4 of the prototype, similar conversions will give the expected model displacements for equivalent model loads.

Both the observed and converted calculated deflections may be found in Table XIII. These values are plotted as shown in Figs. (45, 46). Values of the observed displacements are the average of a number of tests in a series; the plotted points are joined by broken lines.

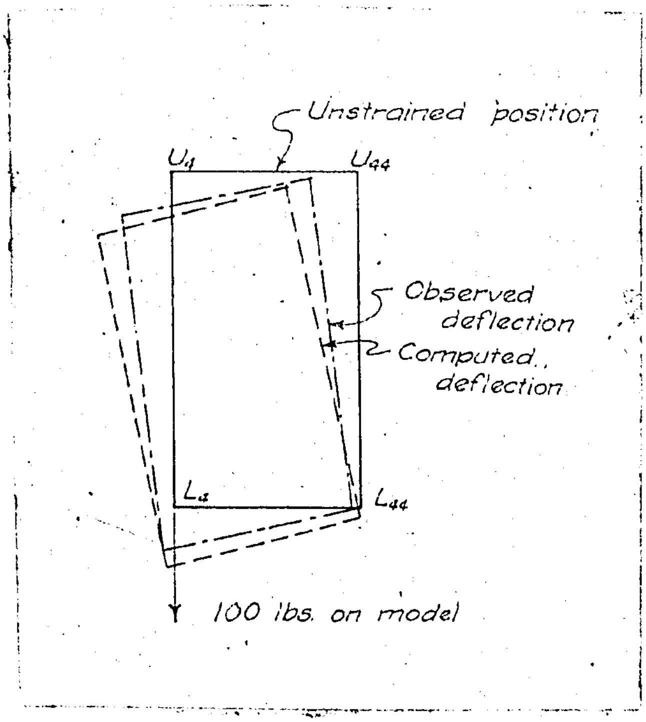


Fig. 48.

The displacement of the transverse model frame as a result of the torque effect of the eccentric load is shown in Fig. (47). The full line represents the unloaded position, the dotted line represents the converted calculated rotation, and the dot-and-dash line represents the observed displacement of the model. This diagram corresponds to 40,000 lbs. prototype and 100 lbs. model loading. For rotations produced by all other test loads see Table XIII.

(b) Stresses.--Stresses in top chords U_3-U_4, bottom L_4-L_5, and in diagonal U_{33}-U_{44} U_5-L_4 of the loaded truss; and in top chord U_{33}-U_{44} of the unloaded truss were measured with Huggenberger Tensometers and the results are compared with those computed in Chapter III for the prototype. In Table XIV a comparison is shown between the model and prototype unit stresses. A few of these are discussed below.

In top chord U_3-U_4 of the prototype, the unit stress

$$S = \frac{-34633}{43.33} = 798 \text{ lbs./sq.in.}$$

which agrees well with the unit stress of - 940 lbs./sq.in. in the corresponding member of the model for a concentrated load of 100 lbs. at L_4, on the assumption that the moduli of elasticity are the same for both the prototype and the model.

In diagonal U_5-L_4 of the prototype, the unit stress,

$$S = \frac{+30182}{29.42} = +1025 \text{ lbs./sq.in.}$$

and this corresponds to the expected unit stress of 1061 lbs./sq.in. in the same member of the model.

In bottom chord L_4-L_5 of the prototype, unit stress

$$S = + \frac{15267}{26.48} = + 576 \text{ lbs./sq.in.}$$

compares favorably with the observed stress of 550 lbs./sq.in.

In top chord U_{33}-U_{44} of the unloaded truss of the prototype for 60,000 lbs. at L_4 gives a unit

$$S = \frac{9949}{43.33} = - 229 \text{ lbs./sq.in.}$$

The unit stress in the same member of the model for 150 lbs. at L_4 was measured to be 219 lbs./sq.in. For other loadings, they vary as loads at L_4; 50 lbs. per joint on both trusses are used for initial readings, and W will be the additional load added at L_4. Measured stresses are based on the average of different tests as recorded by Huggenberger tensometers. Both measured and calculated results are tabulated as shown in Table XIV.

TABLE XIV. UNIT STRESSES ON MODEL

Load at L_4, lbs.	Diagonal U_5-L_4, lbs./sq. in.		Top Chord U_3-U_4, lbs./sq. in.		Bottom Chord L_3-L_4, lbs./sq. in.		Bottom Chord L_4-L_5, lbs./sq. in.	
	Observed value	Calculated value	Observed value	Calculated value	Observed value	Calculated value	Observed value	Calculated value
50	+ 552	+ 513	- 498	- 400	----	----	+ 330	+ 288
100	+1061	+1025	- 940	- 800	+ 440	+ 448	+ 550	+ 576
150	+1540	+1539	-1550	-1200	----	----	+ 775	+ 864
200	+1980	+2050	-2035	-1600	+ 840	+ 896	----	----
250	----	+2560	----	-2000	----	----	----	----
300	----	+3070	-2880	-2400	+1080	+1340	----	----
350	----	+3590	----	-2800	----	----	----	----
400	+3790	+4100	-4015	-3200	+1460	+1790	----	----
450	----	+4610	----	-3600	----	----	----	----
500	----	+5120	-4980	-4000	+1900	+2240	----	----
550	----	+5630	----	-4400	----	----	----	----
600	+5230	+6150	-6160	-4800	+2150	+2690	----	----

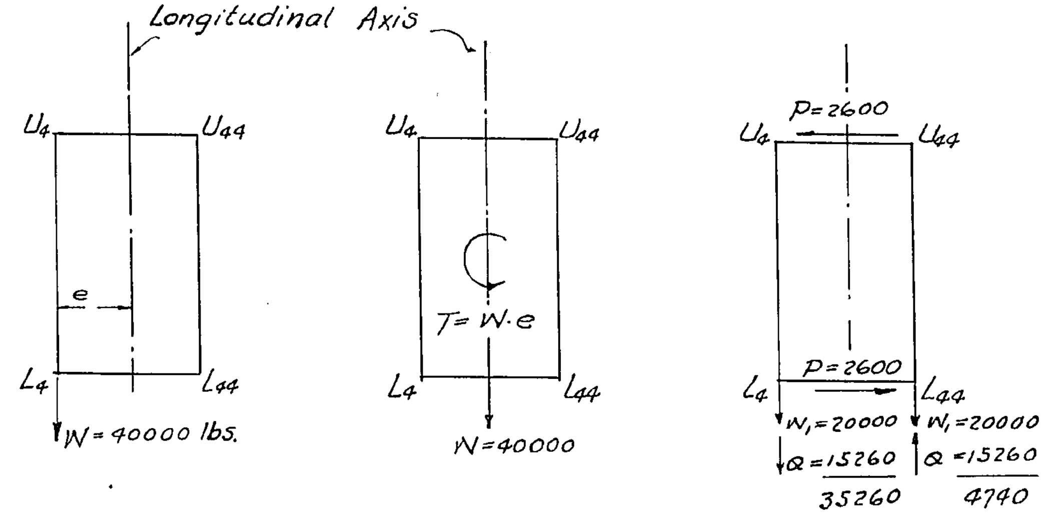
V. SUMMARY

A. INTERPRETATION OF RESULTS

1. Deflections. All horizontal deflections of the four corners of the loaded frame are toward the loaded truss. The two top corners, however, have moved far more than the two lower corners so that the relative motion of the transverse frame is a rotation in the direction of the torque as shown in Fig. (48).

The vertical deflection of the loaded truss and the horizontal deflections of U_4 and U_{44}, although they are low, agree fairly well with the calculated values. This can be explained by the fact that the truss action of the entire bridge is not as ideal as assumed; that is, when the bridge is under eccentric loading, the joints are more rigid than pin-connected, even in the planes of the individual trusses.

The vertical deflection on the unloaded side falls appreciably off from the calculated values; while the horizontal deflections at the two lower corners of the loaded truss are much larger than computed. Although this may be explained in a number of ways, the main reason is that as the bridge rotates, all the freely hanging weights tend to push the bottom laterals toward the loaded truss. Accordingly, the loaded transverse frame is rotated as shown in Fig. (49), when the bridge is under eccentric load. In any case, however, the horizontal deflections at the two lower corners are very small in comparison with other deflections, and their effect is negligible on the problem as a whole.



(a) Applied Load (b) Equivalent Load System (c) Effective Loads

Fig. 49

When an eccentric load W, Fig. 49a, is placed on the bridge its effect may be studied by breaking it into two parts, Fig. 49b; one, an equal load W placed on the longitudinal axis of the bridge, and the other a torque equal to the load W times its eccentricity. For this bridge, Fig. 49c, a load of 40,000 lbs. produces loads W_1 of 20,000 lbs.

intensity at L_4 and at L_{44}; the torque produces a downward load Q of 15,260 lbs. at L_4 and an upward load Q of 15,260 lbs. at L_{44}. The net result produces an effective downward load of 35,260 lbs. at L_4 and an effective downward load of 4,740 lbs. at L_{44}.

A study of these values in terms of the computed and model displacements provides a rational explanation of the general validity of the method and accuracy of theanalysis employed in this investigation.

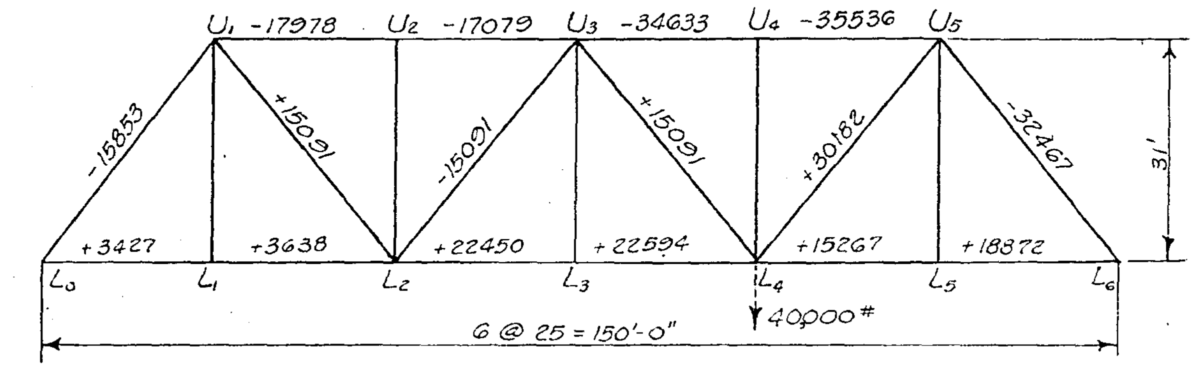
	Effective Load, lbs.		Displacements in $\frac{1}{10000}$ of an inch			
			Model		Computed*	
Joint	Amount	% of total	Amount	% of total	Amount	% of total
L_4	35260	88.1	30.70	89.6	39.1	84.6
L_{44}	4740	11.9	3.57	10.4	7.1	15.4
Total	40000	100.0	34.27	100.0	46.2	100.0

*Converted to model scale by reduction factor.

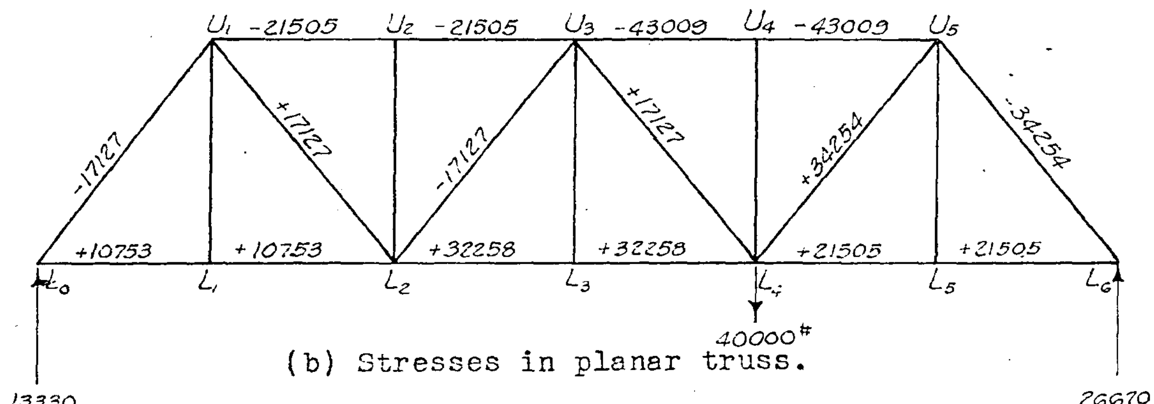
The accompanying table shows the computed effective loads, model vertical displacements and computed vertical displacements at joints L_4 and L_{44}. A comparison between the computed effective loads and the observed vertical displacements at the loaded joint L_4 and the unloaded joint L_{44} indicates almost the same proportional transfer of load, namely, 11.9 per cent by torque calculation and 10.4 per cent by vertical displacement as observed on the model. The load transfer as determined by the computed vertical displacement appears to be 15.4 per cent as compared with 11.9 per cent by torque computation. This close agreement between the factors that may be used to interpret the transfer of load from a loaded to an unloaded truss indicates that the calculated distribution of the torque to the two vertical trusses and the two lateral trusses may be assumed as reasonably correct in theory and accurate in computation.

2. Stresses. An interpretation of the computed stresses may be followed by referring to Fig. 27 on page 74 and to Fig. 50 on page 152. When the bridge is considered as a space structure and a load is applied at L_4, stresses in the loaded vertical truss are as shown in Fig. 50a; and for static equilibrium about each joint, reference should be made to Fig. 27. Stresses in the same truss, when it is considered as a planar structure and is under the same load, are as shown in Fig. 50b. A comparison of these two cases would indicate that axial stresses are greatly reduced when the entire bridge is solved as a space structure, and the greatest reduction is in the bottom chords.

Measured stresses in the diagonal U_5-L_4 agree fairly well with the computed values. This shows that the vertical load on the loaded truss is about the amount as computed. As shown in Table XIV, when the concentrated load increases, the stresses in this diagonal decrease in comparison with the computed values. A few factors could be attributed to this result, but the main reason is that the actual torque created by the eccentric load is not a pure torque as was assumed. This torque decreases as the bridge rotates.



(a) Stresses in space structure
as computed in III.
40000 lbs. on Bridge.



(b) Stresses in planar truss.

Fig. 50.

Comparison of Bridge Stresses.

3. Estimate of Errors Involved. Errors involved in the whole problem are attributed to the ideal assumptions; to the distribution of load in the mathematical computations; to the design of the model; to the construction of the model; and to the observations of measurements.

Thus in the computations, as noted before, the division of the equivalent load system could be made in different ways, and each system will not affect any of joints aside from the loaded transverse frame but to a certain extent it affects members at the local joint. This is well explained in the Principle of St. Venant.

In the design of the model, certain percentages of error have been allowed in proportioning sections and their structural properties. No error is above 10% and generally much less than this amount. But all errors as a whole are compensating for the entire structure rather than accumulating, for some of the properties are more and some less than desired.

Tensometers for stress measurements could be read to 1/10000 of an inch for unit strain. That means, for E equal to 27 million lbs. per sq.in., these instruments can be read to the degree of 2700 lbs per sq. in. for unit stresses. Stresses less than this are a fair guess. Also rough surfaces at the brazed joints will add more or less errors for deflections, since 1/10000 of an inch deflection is rather too small a distance not to be affected by workmanship.

When all these factors are summed up, measurements made on the model to compare with the calculated values for a prototype are rather satisfactory, and in fact better than expected.

B. CONCLUSIONS

With a finite number of measurements, the author prefers not to make conclusions that might be altogether too definite. Taking all factors into consideration, however, he feels that he is justified to make these statements.

1. Tests on the model have definitely demonstrated that it is not only possible to design a space model from a definite prototype, but also that such a model is quite practical. Also, a model can be produced at a very low cost in comparison with what it would cost to make a miniature reproduction; and the structural behavior of the model can be made quite similar to that of the prototype.

2. Tests on the model have definitely proved that all transverse frames are restrained to a high degree and are able to transmit torsion.

3. The transverse frames rotate in the direction of the torque.

4. The distribution of the torque among the trusses composing the bridge can be determined fairly accurately by the method used in Chapter III.

5. The effect of torsion is distributed among the vertical and lateral trusses in proportion to their relative stiffness. In other

words, where there is an eccentric load on the floor beam, the effective loads on the two vertical trusses are not the simple beam reactions of the floor beam.

6. In the case of extreme eccentricity, when the applied load is in the plane of one of the vertical trusses, the unloaded truss carries a portion of the applied load.

7. The stresses in all members of the vertical truss which carries the larger portion of an eccentrically applied load are smaller than the stresses obtained by the usual method of distributing a load eccentrically placed on the floor beam.

8. Stress analysis which does not take into account the transfer of torque through the transverse frame leads to erroneous interpretations. The result from this method of analysis, which has appeared in recent literature and is again presented in Appendix A, does not check with the model tests in Chapter IV. On the other hand, the result from the computations made in Chapter III, which have taken into account the transfer of torque effect through the transverse frame, does check with the model tests.

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VII. ACKNOWLEDGMENTS

The author wishes to express his indebtedness to Professors A. H. Fuller and Frank Kerekes for their valuable directions, advice and encouragement to make this theoretical analysis as well as the model design.

He also feels grateful to Mr. R. W. Breckenridge of the Mechanical Engineering Department for his patience and interest in making the bridge model; and to Messrs. George Fagels, Jr. and B. A. Moore, both of the Engineering Experiment Station, for their help in making the stress and deflection measurements.

VIII. VITA

The author was born in 1909 near Peiping in Northern China. At six years of age, he started his primary school education; and from 1920 to 1925, he was through with his high school training at Ts'un Shih, near Peiping. In 1925, when the former Tsing Hua College at Peiping was reorganized on university basis, he entered there for college education.

His equal interest in literature and natural sciences made a decision as to his further study difficult, but viewing the actual need of his country, he finally determined to pursue an engineering course.

After graduation in 1929, he went to Manchuria, where he spent one year in teaching at an engineering college. Late in 1930, he was sent by his provincial government to the United States for more extensive study.

He spent his first year and a half at the graduate school of Cornell University, where he received the Degree of Master of Science under Professor L. C. Urquhart in June, 1932. In the fall of 1932, he attended the University of Minnesota in pursuit of more graduate work under Professor J. I. Parcel; and in the spring of 1933, he transferred to Iowa State College. His graduate work, however, was interrupted once in the fall of 1933, when he went to Chicago to practice with the Strauss Engineering Corporation. His short stay with this company, in making practical computations and designs, has provided him with more practical knowledge.

He returned to Iowa State College in January, 1934, and since then has devoted his entire interest to researches in structures. On his return to China he hopes to exert all of his abilities for the welfare of his fatherland.

Ho-Cheng Chai

Ames, Iowa, U. S. A. March, 1935

APPENDIX A

1. Stress Analysis of a Bridge Frame on the Assumption that it is Completely Pin-Connected. The conventional assumption used to solve the bridge as a space structure is that all joints are pin-connected in all directions, and the degree of redundancy is determined from the equation,

$$n = 3m - 6$$

where n is the number of members required to make the structure stable, and m is the necessary number of space joints.

When the structure is redundant, the solution is obtained just as for planar structures by a direct application of the strain-energy method.

Before the author decided to make the solution given in Chapter III, and before he designed a model to check up the computed results, he solved the problem by applying the method outlined above. The bridge dimensions and the L/EA values of the members are about the same as those for the bridge analyzed in Chapter III. Because of the tedious work involved in solving redundant members, only single diagonal bracing is used in the lateral trusses. The portals, however, are assumed to be braced with full diagonals.

The bridge is loaded with 15,000 lbs. at L_4 as shown in Fig. (51).

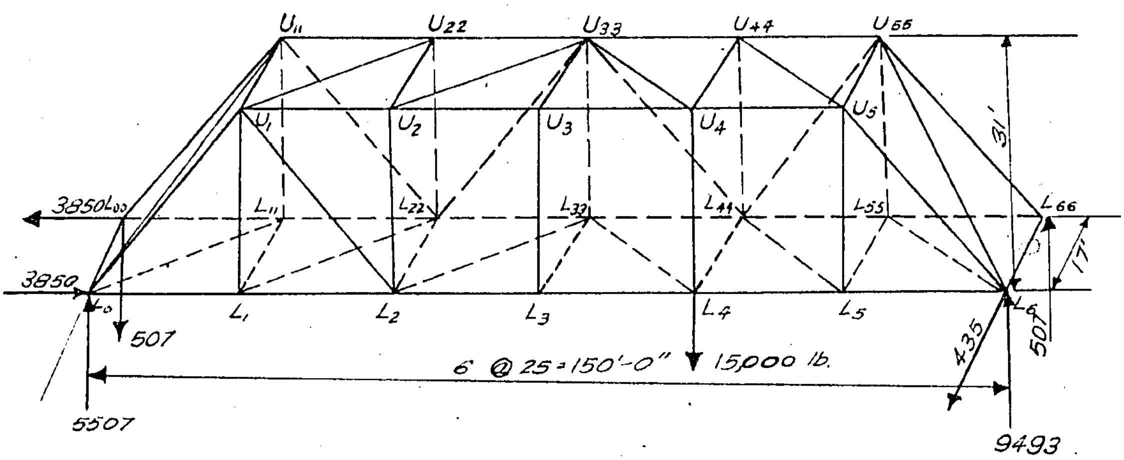


Fig. 51.

Bridge Structure
with
Single Lateral Bracings.

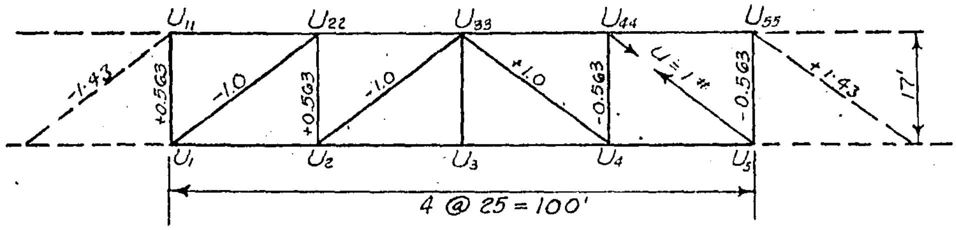
The number of reactions, as usually provided for ordinary bridges, are shown in the figure. The number of members in the space structure:

2 vertical trusses at 21	42
top lateral, not including top chords	9
bottom lateral, not including bottom chords	13
transverse bracings	2
Total number of members	66
and total reactions	8
Total number of unknowns	74

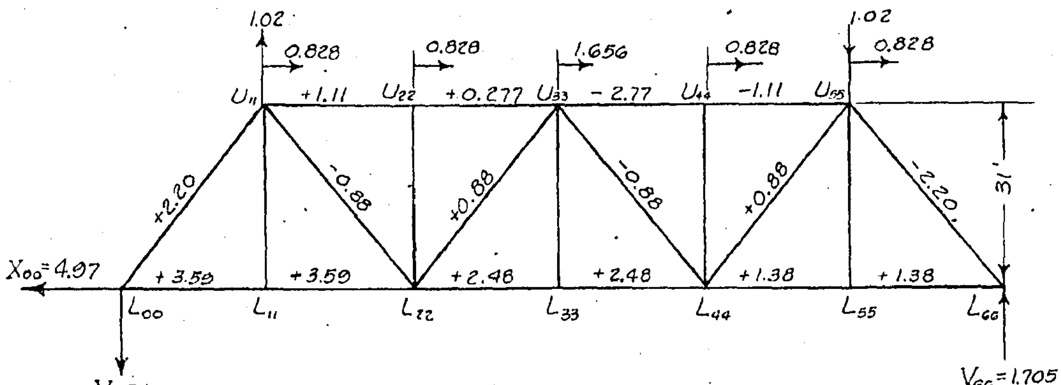
The number of space joints is 24, and the number of equations by considering the static of equilibrium is 72. Therefore, the degree of redundancy of the entire bridge is 2.

Assume $U_{44}-U_5$, and $L_{55}-L_6$ as redundant members and remove them from the bridge. The stresses S' in the bridge are those due to the concentrated load at L_4 only; and in this case S' exists in the loaded vertical truss only after the redundancies have been removed.

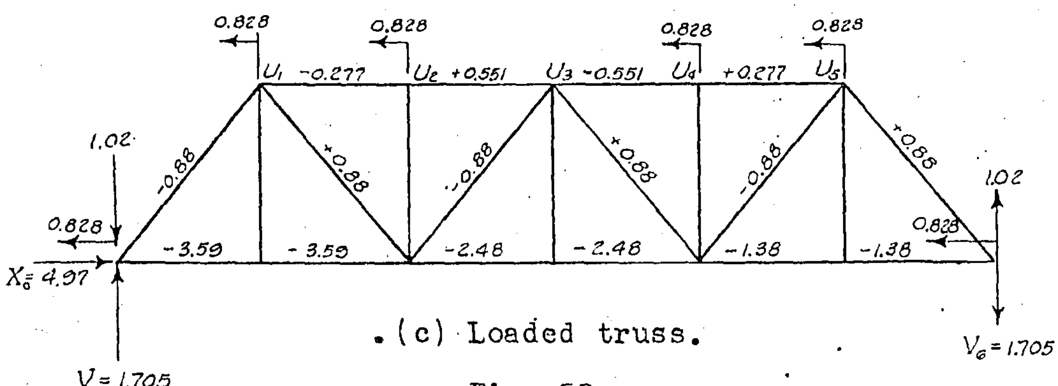
The u_1 stresses are those due to one pound tension in redundant member $U_{44}-U_5$, and the horizontal components of the top diagonals along the direction of top chords on the vertical trusses as shown



(a) Top diagonals and struts.



(b) Unloaded truss.



(c) Loaded truss.

Fig. 52.
Stresses in Bridge due to unit load in redundant member U_{44}-U_5.

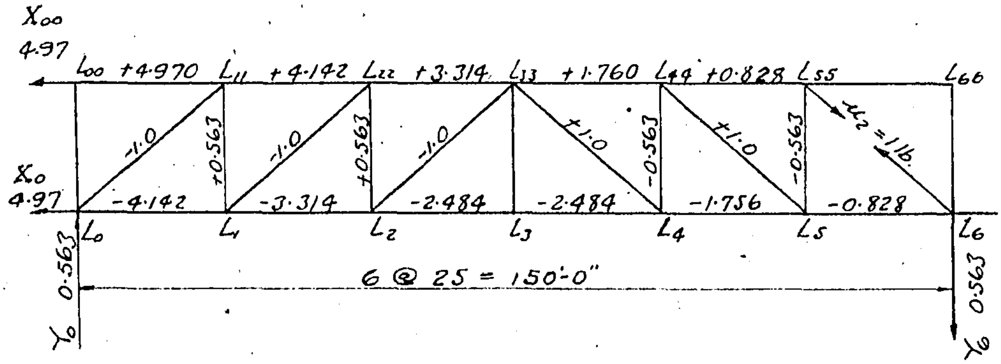


Fig. 53.

Stresses in Bridge due to
unit load in redundant member
L_{55}-L_6.

in Fig. (52). The u_2 stresses exist only in the bottom lateral truss, and these are shown in Fig. (53). If R_1 and R_2 are the axial stresses in the redundant members U_{44}-U_5 and L_{55}-L_6 respectively under the external load, then the two following equations will be true:

$$\frac{\partial W}{\partial R_1} = 0,$$

$$- 924,200 + 1400.88R_1 + 854.20R_2 = 0 \qquad (1)$$

$$\frac{\partial W}{\partial R_2} = 0,$$

$$- 903,800 + 854.2R_1 + 1674.2R_2 = 0 \qquad (2)$$

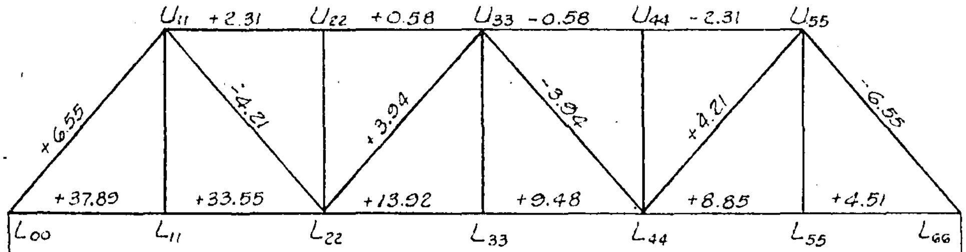
Solving,

$$R_1 = + 477 \text{ lbs., and } R_2 = + 298 \text{ lbs.}$$

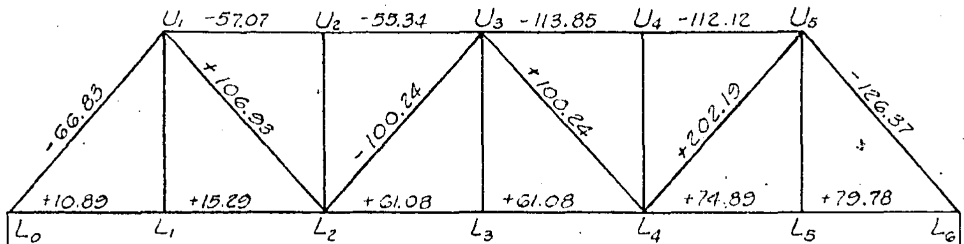
Final stresses in the entire bridge are obtained by substitution as tabulated in Tables XV to XIX inclusive.

It is of particular interest, to observe that the top lateral truss will bend like an S curve with the portion near the load bending inward, and the left portion of the bridge bending outward. In the bottom lateral truss, it also develops an S curve, only opposite to that in the top lateral truss. The cause for the development of an S curve is due to the heavy compressions in the top chords around U_4 and heavy tensions in those bottom chords around L_4, and due to the assumption that all joints are pin-connected in all directions.

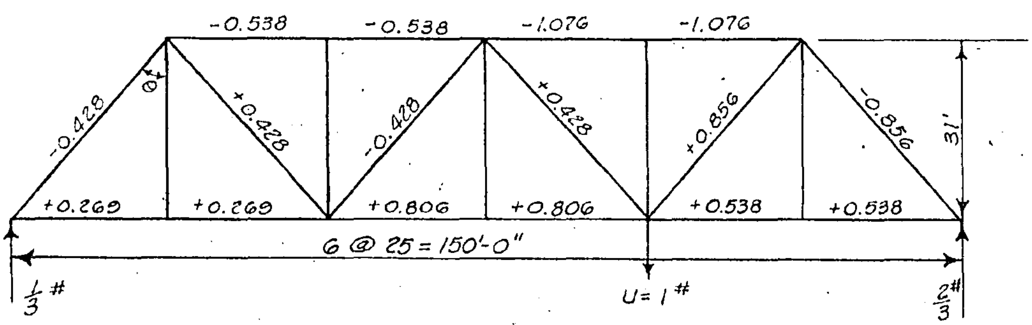
2. Displacements of the Bridge Structure. The displacements of the structure can be either computed or found from displacement



(a) L/A values in Truss TT.



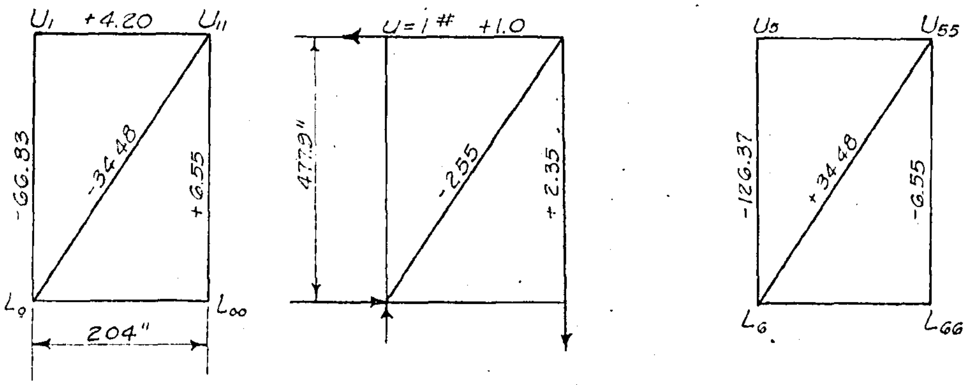
(b) L/A values in Truss T.



(c) 'u' stress in vertical truss.

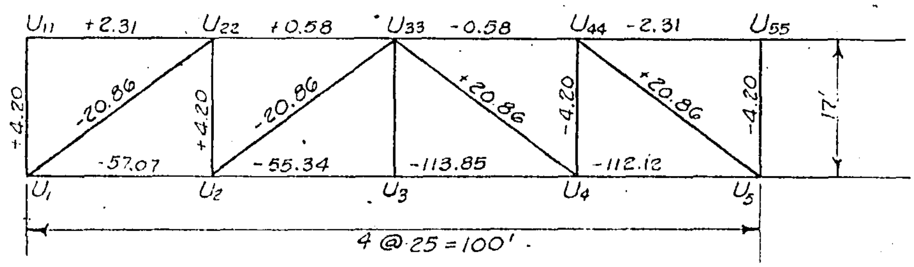
Fig. 54.

Data for vertical truss deflections with 15000 lbs. at L_4.

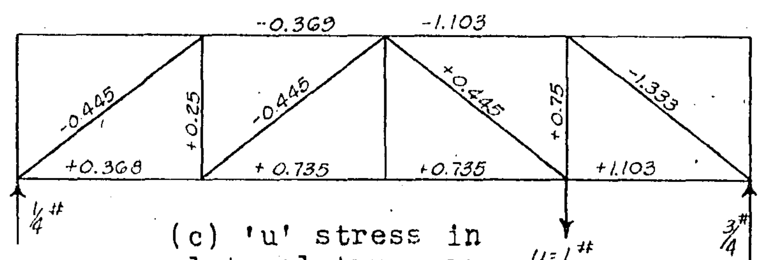


$\frac{L}{A}$ values. 'u' stress $\frac{L}{A}$ values.

(a) Portals with full diagonals.



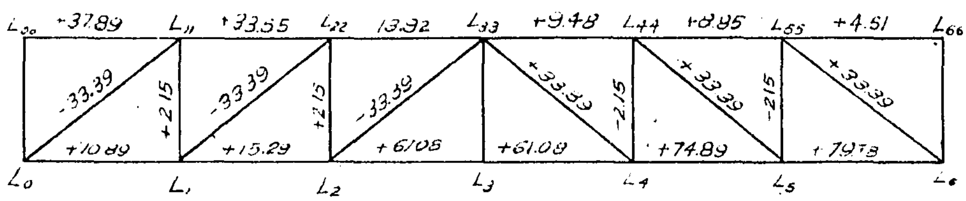
(b) L/A values for lateral truss.



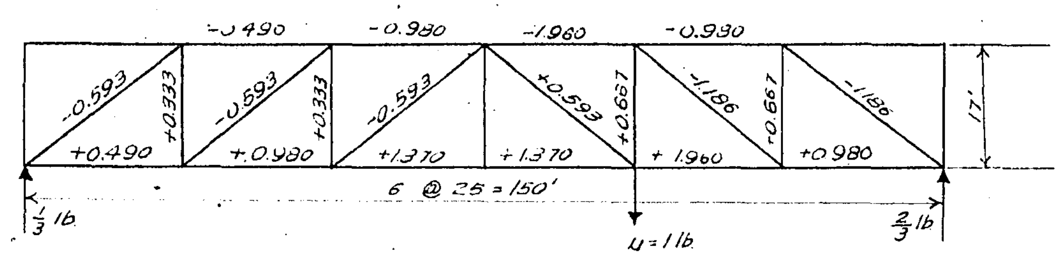
(c) 'u' stress in lateral truss as simply supported.

Fig. 55.

Data for top lateral deflections with 15000 lbs. at L_4.



(a) L/A values.



(b) 'u' stress in bottom lateral truss as simply supported.

Fig. 56.
Data for bottom lateral deflections with 15000 lbs. at L_4.

TABLE XV. DEFLECTION AT L_4 OF THE LOADED VERTICAL TRUSS

Member	$\frac{L}{A}$	Final Stresses S, Lbs.	u	$\frac{SL}{EA}$, Kips
U_1-U_2	7	- 8153	- 0.538	+ 30.70
U_2-U_3	7	- 7906	- 0.538	+ 29.77
U_3-U_4	7	-16264	- 1.076	+122.50
U_4-U_5	7	-16017	- 1.076	+120.64
L_0-L_1	11	+ 990	+ 0.269	+ 2.93
L_1-L_2	11	+ 1390	+ 0.269	+ 4.11
L_2-L_3	6	+10180	+ 0.806	+ 49.23
L_3-L_4	6	+10180	+ 0.806	+ 49.23
L_4-L_5	11	+ 6808	+ 0.538	+ 40.29
L_5-L_6	11	+ 7253	+ 0.538	+ 42.92
U_1-L_0	10	- 6683	- 0.428	+ 28.60
U_1-L_2	16	+ 6683	+ 0.428	+ 45.76
U_3-L_2	15	- 6683	- 0.428	+ 42.90
U_3-L_4	15	+ 6683	+ 0.428	+ 42.90
U_5-L_4	16	+12637	+ 0.856	+173.07
U_5-L_6	10	-12637	- 0.856	+108.17
U_1-L_1	15	-----	-----	-----
U_2-L_2	30	-----	-----	-----
U_3-L_3	15	-----	-----	-----
U_4-L_4	30	-----	-----	-----
U_5-L_5	15			

E = 1 Σ = +933.72

Vertical deflection of L_4 = 933.72 downward

TABLE XVI. DEFLECTION AT L_{44} OF THE UNLOADED VERTICAL TRUSS

	Members	$\frac{L}{A}$	Final Stresses S, lbs.	u, in lbs.	$\frac{SL}{EA}$, Kips
Top Chords	$U_{11}-U_{22}$	7	+ 330	- 0.538	- 1.24
	$U_{22}-U_{33}$	7	+ 83	- 0.538	- 0.31
	$U_{33}-U_{44}$	7	- 83	- 1.076	+ 0.62
	$U_{44}-U_{55}$	7	- 330	- 1.076	+ 2.48
Bottom Chords	$L_{00}-L_{11}$	11	+3445	+ 0.269	+10.19
	$L_{11}-L_{22}$	11	+3050	+ 0.269	+ 9.02
	$L_{22}-L_{33}$	6	+2320	+ 0.806	+11.22
	$L_{33}-L_{44}$	6	+1580	+ 0.806	+ 7.64
	$L_{44}-L_{55}$	11	+ 805	+ 0.538	+ 4.76
	$L_{55}-L_{66}$	11	+ 410	+ 0.538	+ 2.43
Diagonals	$U_{11}-L_{00}$	10	+ 655	- 0.428	- 2.80
	$U_{11}-L_{22}$	16	- 263	+ 0.428	- 1.80
	$U_{33}-L_{22}$	15	+ 263	- 0.428	- 1.69
	$U_{33}-L_{44}$	15	- 263	+ 0.428	- 1.69
	$U_{55}-L_{44}$	16	+ 263	+ 0.856	+ 3.60
	$U_{55}-L_{66}$	10	- 655	- 0.856	+ 5.61
Verticals	$U_{11}-L_{11}$	15	----	----	----
	$U_{22}-L_{22}$	30	----	----	----
	$U_{33}-L_{33}$	15	----	----	----
	$U_{44}-L_{44}$	30	----	----	----
	$U_{55}-L_{55}$	15	----	----	----

E = 1 $\quad\quad$ Σ = +48.04

Vertical deflection of L_{44} = 48.04 downward

TABLE XVII. PORTAL DEFLECTIONS

	Member	$\frac{L}{A}$	Final Stresses S, lbs.	u	$\frac{SuL}{EA}$, Kips
Left Portal	$U_1 - U_{11}$	25	+ 168	+ 1.0	+ 4.20
	$L_0 - L_{00}$	8	-----	-----	-----
	$U_1 - L_0$	10	-6683	-----	-----
	$U_{11} - L_{00}$	10	+ 655	+ 2.35	+15.40
	$U_{11} - L_0$	80	- 431	- 2.55	+87.92
		E = 1			Σ = +107.52
Right Portal	$U_5 - U_{55}$	25	- 168	+ 1.0	- 4.20
	$L_6 - L_{66}$	8	-----	-----	-----
	$U_5 - L_6$	10	-12637	-----	-----
	$U_{55} - L_{66}$	10	- 655	+ 2.35	-15.40
	$U_{55} - L_6$	80	+ 431	- 2.55	-87.92
		E = 1			Σ =107.52

Left Portal:

Deflection at U_1 = +107.52

Deflection at U_{11} = (107.52 - 4.2) = +103.32

Right Portal:

Deflection at U_5 = -107.52

Deflection at U_{55} = -103.32

TABLE XVIII. DEFLECTION AT U_4, U_{44} OF THE TOP LATERALS

	Member	$\frac{L}{A}$	Final Stresses S, lbs.	u	$\frac{SuL}{EA}$, Kips
Top Chords	U_{11}-U_{22}	7	+ 330	----	----
	U_{22}-U_{33}	7	+ 83	-0.368	- 0.21
	U_{33}-U_{44}	7	- 83	-1.103	+ 0.64
	U_{44}-U_{55}	7	- 330	----	----
	U_1 -U_2	7	- 8153	+0.368	-21.00
	U_2 -U_3	7	- 7906	+0.735	-40.67
	U_3 -U_4	7	-16264	+0.735	-83.68
	U_4 -U_5	7	-16017	+1.103	-123.67
Head Struts	U_{11}-U_1	25	+ 168	----	----
	U_{22}-U_2	25	+ 168	+0.25	+ 1.05
	U_{33}-U_3	25	----	----	----
	U_{44}-U_4	25	- 168	+0.75	- 3.15
	U_{55}-U_5	25	- 168	----	----
Top Diagonals	U_1 -U_{22}	70	- 298	-0.445	+ 9.28
	U_2 -U_{33}	70	- 298	-0.445	+ 9.28
	U_{33}-U_4	70	+ 298	+0.445	+ 9.28
	U_{44}-U_5	70	+ 298	-1.333	-27.81

E = 1 Σ = -270.66

Deflection at U_4 due to lateral truss, but not portals, = -270.66

Deflection at U_{44} due to lateral truss, but not portals, = (-270.66 - 4.20) = - 274.86

TABLE XIX. DEFLECTIONS AT L_4, L_{44} OF THE BOTTOM LATERALS

	Member	$\frac{L}{A}$	Final Stresses S, lbs.	u	$\frac{SuL}{EA}$, Kips
Bottom Chords	$L_{00}-L_{11}$	11	+ 3445	----	----
	$L_{11}-L_{22}$	11	+ 3050	- 0.490	- 16.44
	$L_{22}-L_{33}$	6	+ 2320	- 0.980	- 13.64
	$L_{33}-L_{44}$	6	+ 1580	- 1.960	- 18.58
	$L_{44}-L_{55}$	11	+ 805	- 0.980	- 8.67
	$L_{55}-L_{66}$	11	+ 410	----	----
	L_0-L_1	11	+ 990	+ 0.490	+ 5.34
	L_1-L_2	11	+ 1390	+ 0.980	+ 14.98
	L_2-L_3	6	+10180	+ 1.370	+ 83.68
	L_3-L_4	6	+10180	+ 1.370	+ 83.68
	L_4-L_5	11	+ 6808	+ 1.960	+146.78
	L_5-L_6	11	+ 7253	+ 0.980	+ 78.18
Floor Beams	$L_{00}-L_0$	8	----	----	----
	$L_{11}-L_1$	8	+ 269	+ 0.333	+ 0.71
	$L_{22}-L_2$	8	+ 269	+ 0.333	+ 0.71
	$L_{33}-L_3$	8	----	----	----
	$L_{44}-L_4$	8	- 269	+ 0.667	- 1.43
	$L_{55}-L_5$	8	- 269	+ 0.667	- 1.43
	$L_{66}-L_6$	8	----	----	----
Bottom Diagonals	L_0-L_{11}	70	- 477	- 0.593	+ 19.80
	L_1-L_{22}	70	- 477	- 0.593	+ 19.80
	L_2-L_{33}	70	- 477	- 0.593	+ 19.80
	$L_{33}-L_4$	70	+ 477	+ 0.593	+ 19.80
	$L_{44}-L_5$	70	+ 477	- 1.186	- 39.60
	$L_{55}-L_6$	70	+ 477	- 1.186	- 39.60

E = 1 $\qquad\qquad$ Σ = +353.87

diagrams. Since the distortion of the loaded transverse frame at L_4 has been specifically studied in this problem, the displacements for those four corners are calculated in this case. Taking each truss separately, the deflection at any point is found from the equation,

$$d = \sum \frac{SuL}{EA}$$

Thus 1 lb. is applied at L_4 and L_{44} and the deflections are found as tabulated in Tables XV to XIX. For the top lateral truss, the u stress for finding deflections should be computed by taking the entire top lateral truss including the fully braced portals as one unit. For simplicity of computations, however, little error will be involved by considering the entire top lateral truss as made up of three trusses: lateral truss, and the two fully braced portals. Then deflections at U_4, and U_{44} are found by considering the top lateral truss as simply supported, and then later deflections due to portals are added.

One point to be noted is that the vertical deflections at U_4 and U_{44} are the same as at L_4 and L_{44} respectively. Horizontal deflection at U_{44} is different from that at U_4 by the deformation of the head strut U_4-U_{44}. The same reasoning applies to the horizontal deflections at L_4, L_{44} in the bottom lateral truss. Distortion of the transverse loaded frame as calculated above is then scaled as shown in Fig. (57).

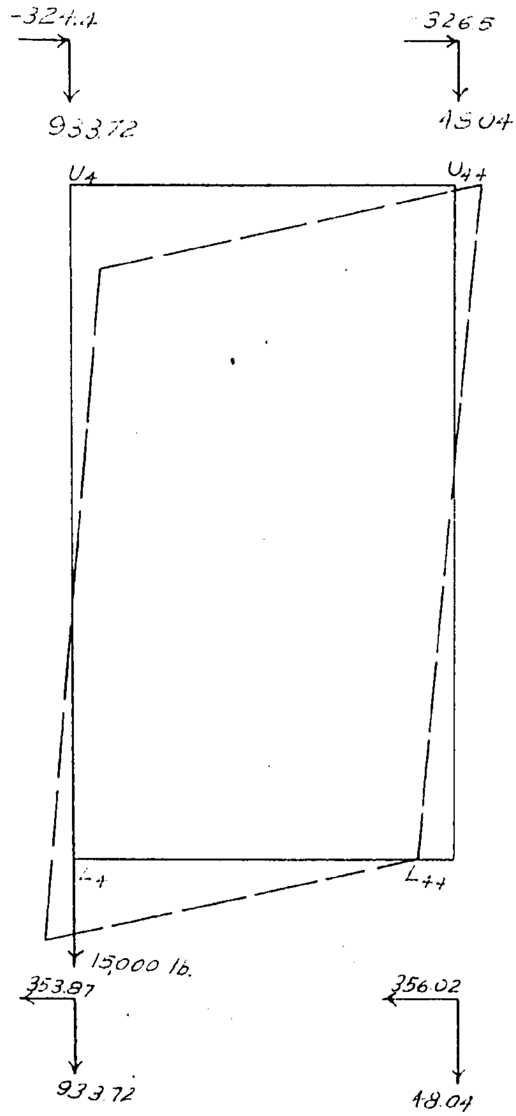


Fig. 57.

Displacement of Transverse Frame U_4-U_{44}.

Horizontal deflection at U_4:

due to portal settlements,

$$= \frac{-107.52}{2} = -53.76$$

due to lateral truss action,

$$= -270.66$$

$$\text{Total} \quad = -324.42$$

Horizontal deflection at U_{44}:

due to portal settlements,

$$= -\frac{103.32}{2} = -51.66$$

due to lateral truss action,

$$= -274.86$$

$$\text{Total} \quad = -326.52$$

Bottom laterals:

Horizontal deflection at L_4 = + 353.87

Horizontal deflection at L_{44} = 353.87 - Effect of Member

$L_{44} - L_4$ when u is applied at L_{44}

$$= (353.87 + 2.15) = +356.02$$

It is quite evident that the rotation of the loaded transverse frame as against the direction of the torque created by the eccentric load is illogical. From another point of view, the solution is correct only for the assumption that all joints are pin-connected in space, and the effect of torsion, therefore, could not be transmitted through the loaded transverse frame, which is assumed as a pin-connected paralellogram.

The same result has been obtained by W. Bergfelder (3). Actual measurements made by R. Bernhard on a series of bridges in Europe, however, have shown that this solution is far from the fact.

APPENDIX B

True stiffness of the Portal Strut. On account of the knee braces which are generally used in bridge portals, the portal struts are greatly stiffened. The true stiffness of the portal struts can be best determined by the method of Column Analogy developed by Professor Hardy Cross (9).

Using this method, the indeterminate moment at any section of a beam to preserve its continuity can be found from a formula similar to the one used for finding the fiber stress of an eccentrically loaded column.

In this method of analysis it is first necessary to plot the $\frac{1}{EI}$ ordinates for the original beam. This diagram is called the analogous column section. For this analogous section,

$$m_i = \frac{P}{A} \pm \frac{Mc}{I_A}$$

m_i = the indeterminate moment at any section of the original beam.

P = the load on the analogous column section, in terms of angle of rotation caused by the applied loads.

A = the area of the analogous column section which is equal to $\int_0^L \frac{ds}{I_o}$, where I_o is for the original beam.

M = the moment produced by the eccentric load about the centroid of the analogous section.

c = the fiber distance from the centroid of the analogous section.

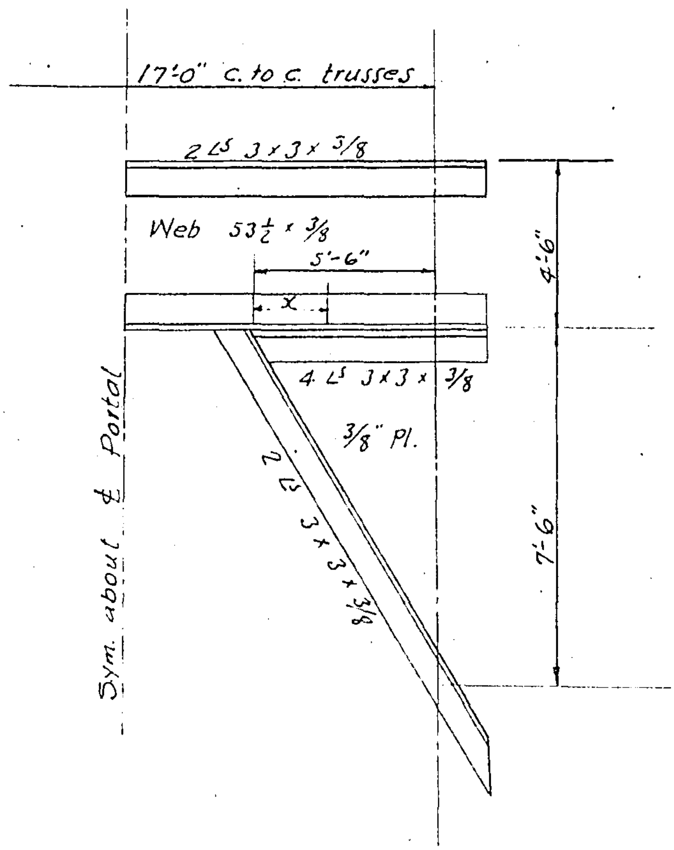


Fig. 58.

Detail Section of Portal Strut.

I_A = the moment of inertia of the analogous section area through its centroid.

Also, the stiffness of any beam, hinged at one end and fixed at the other end, may be defined as the moment required to produce unit rotation at the hinged end.

For the section of the portal strut as shown in Fig. (58), the moment of inertia of different sections of the strut is first found.

The values for the moment of inertia are shown in Table XX for sections located as distances x as shown in Fig. (58). In this table I_s is the moment of inertia of the constant portion of the head strut and I_b is the moment of inertia of the bracket portion, both being taken about the centroid of the section.

Section properties are then tabulated as follows:

TABLE XX. SECTION PROPERTIES OF PORTAL STRUT

x, in.	Centroid	I_s, in.^{4}	I_b, in.^{4}	I_o, in.^{4}
0	0.96	10573	7.2	10580
10	11.40	13536	7620	21156
20	16.60	18396	13130	31526
30	22.10	24446	21080	45526
40	27.80	37546	31380	68926
50	33.70	42946	44810	87756
60	39.60	55247	63100	118346
66	43.20	63746	73700	137446

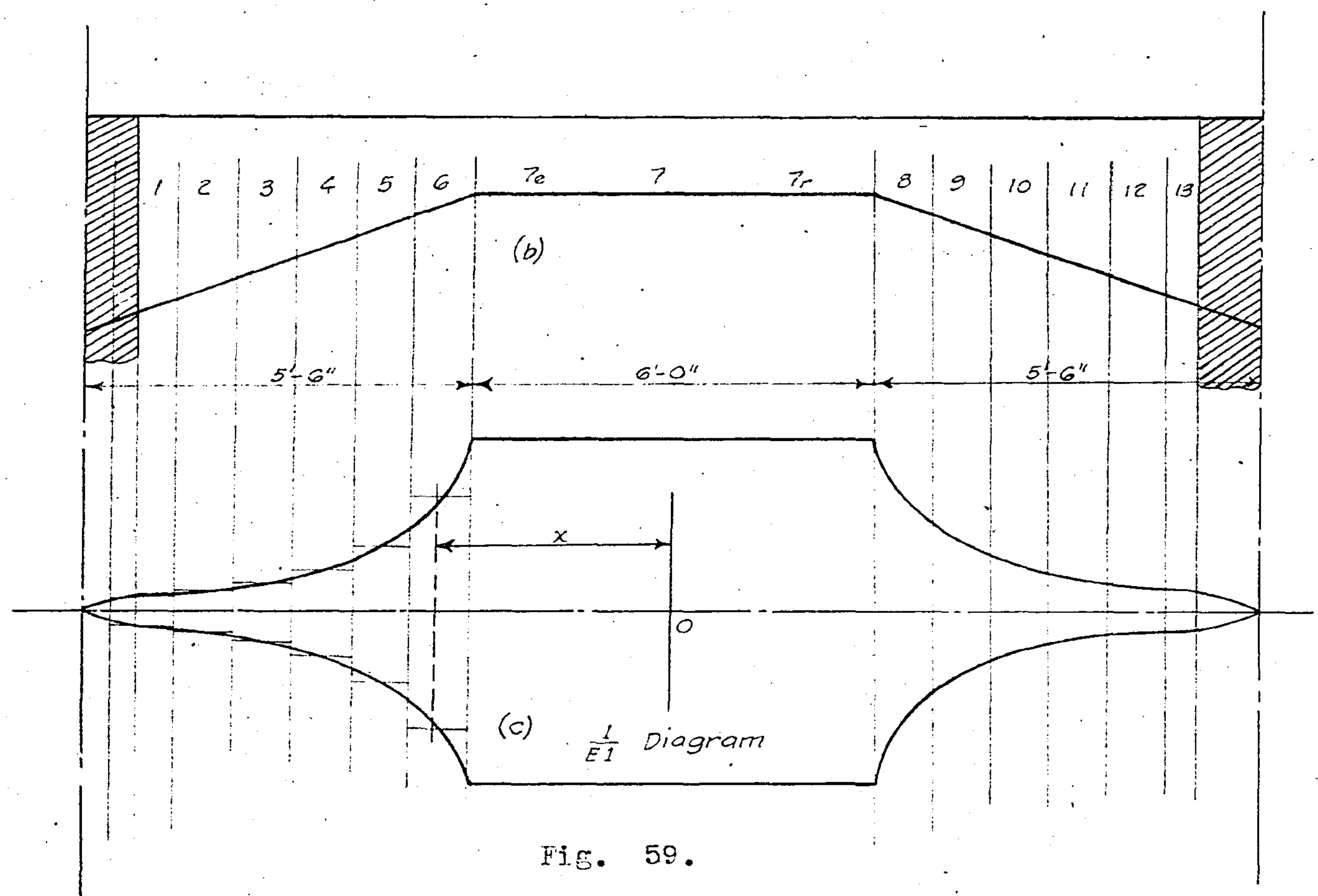


Fig. 59.

True Stiffness of Portal Strut.

The 1/EI diagram is plotted as shown in Fig. (59). In the diagram, due to the small magnitudes of 1/I values, 1/118346 at the distance of x equal to 60 in., is used as a unit. Moment of inertia of the 1/I diagram with respect to its own centroid, 0, is computed by dividing up the entire diagram into small strips as shown in Fig. (59). On account of symmetry, only one-half of the section is needed.

TABLE XXI. SECTION PROPERTIES OF THE PORTAL STRUT

Section	Length L, in.	Width, 1/I	a = L/I	x from centroid, 0	ax^2	i_o, $\frac{1/I(L)^3}{12}$	$ax^2 + i_o$
1	6	1.3	7.8	89	61000	23.4	61020
2	10	1.6	16	81	114800	133.5	114930
3	10	1.9	19	71	95800	158.5	95960
4	10	3	30	61	111500	250	111750
5	10	4.7	47	51	122200	392	122590
6	10	8.7	87	41	146000	725	146730
7	36	11.2	430	18	139400	43400	182800
				Σ=	636.8		Σ=835780
The other half section has the same values.					636.8		835780
					A = 1273.6		I=1671560

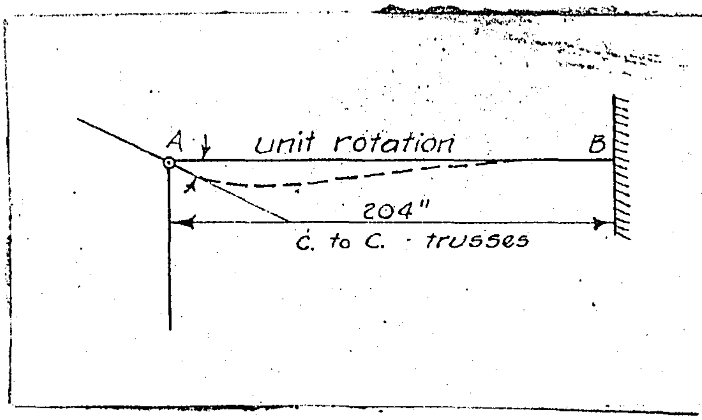


Fig. 60

The load of the angle of rotation,

$$P = 1 \text{ applied at } A.$$

Area of the analogous section

$$A = 1273.6$$

$$M = Pe = 102$$

Moment at A required to produce unit rotation at A while B is fixed,

$$\text{Stiffness } AB = \frac{P}{A} + \frac{Me}{I}$$

$$= \left(\frac{1}{1273.6} + \frac{102 \times 102}{1671560}\right) 118360$$

$$= 829.01 \text{ in.}^3$$

The moment of inertia of the portal strut at mid-section is 10546 in.^{4}; if this section were maintained throughout its entire length, then its stiffness would be,

$$S_{AB} \text{ for uniform section is}$$

$$\frac{4I}{l} = \frac{4 \times 10546}{204} = 207 \text{ in.}^3$$

Thus thq knee braces have increased the stiffness of the portal strut from 207 in.^{3} to 829.01 in.^{3}.